Dynamic Energy Management for the Smart Grid With Distributed Energy Resources

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***Abstract*—Due to its salient features including real time pricing and distributed generation, the smart grid (SG) poses great challenges for energy management in the system. In this paper, we investigate optimal energy management for the SG, taking into consideration unpredictable load demands and distributed energy resources. Both delay intolerant (DI) and delay tolerant (DT) load demands are studied. We aim to optimally schedule the usage of all the energy resources in the system and minimize the long-term time averaged expected total cost of supporting all users' load demands. In particular, we first formulate an optimization problem, which turns out to be a time-coupling problem and prohibitively expensive to solve. Then, we reformulate the problem using Lyapunov optimization theory and develop a dynamic energy management scheme that can dynamically solve the problem in each time slot based on the current system state only. We are able to obtain both a lower and an upper bound on the optimal result of the original optimization problem. Furthermore, in the case of both DI and DT load demands, we show that DT load demands are guaranteed to be served within user-defined deadlines. Extensive simulations are conducted to validate the efficiency of the developed schemes.**

***Index Terms*—Delay, dynamic energy management, energy storage device, renewable energy resource, smart grid.**

NOMENCLATURE

$e_i(t)$	User i's available renewable energy resource in time slot t.
$l_i(t)$	User i's delay intolerant (DI) load demand in time slot t.
$T_i(t)$	User i's delay tolerant (DT) load demand in time slot t.
$r_i^l(t)$	User i's renewable energy used to satisfy its DI load demand in time slot t.
$r_i^g(t)$	User i's renewable energy sold to the power grid in time slot t.
$c_i^r(t)$	User i's renewable energy used to charge its energy storage device in time slot t.
$c_i^g(t)$	Grid energy for charging user i's energy storage device in time slot t.
$C_i(t)$	Total energy for charging user i's energy storage device in time slot t.
$d_i^l(t)$	Discharged energy from user i's energy storage device for satisfying its DI load demand in time slot t.
$d_i^g(t)$	Discharged energy from user i's energy storage device sold to the grid in time slot t.
$d_i^q(t)$	Discharged energy from user i's energy storage device for satisfying its DT load demand in time slot t.
$D_i(t)$	Total discharged energy from user i's energy storage device in time slot t.
$B_i(t)$	Energy level of user i's energy storage device in time slot t.
$X_i(t)$	Adjusted energy level of user i's energy storage device in time slot t.
$G_i(t)$	Total energy user i draws from the power grid in time slot t.
$Q_i(t)$	User i's DT load queue size in time slot t.
$Z_i(t)$	User i's virtual DT load queue size in time slot t.
$\mu_i^{\max}$	Maximum delay for satisfying user i's DT load demand.
$U(t)$	Total energy generation cost in time slot t.
$R(t)$	Utility company's available renewable energy resource in time slot t.

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I. INTRODUCTION

LARGELY underutilized generation capacity and high transmission losses are two major sources of system inefficiency in traditional power grids. Recent studies show that the average utilization of the generation capacity is below 55% [1] and 7% of generated energy is lost due to transmission inefficiencies [2]. In particular, since enough generation capacity is required to be available to meet peak-hour load demand plus a security margin, some power plants are largely unused or underutilized. Besides, energy users are usually several hundreds of miles away from power plants, which inevitably results in a significant amount of energy loss due to transmission inefficiencies. Moreover, overall electricity consumption is projected to increase by about 14% in the next 20 years [3], which will require a big investment to expand the generation and transmission capacity to accommodate the new demand.

Recently, the smart grid (SG) has been proposed as a new electrical grid to modernize current power grids and enhance its efficiency, reliability, and sustainability. Particularly, in the SG, a digital communication network is deployed to enable two-way communications between users and system operators. It thus makes it possible to shape the users' load demand curves by means of *demand response (DR)* strategies, i.e., to encourage customers to change their usual electricity consumption patterns by incentives [4]. One such strategy is real time pricing (RTP), in which system operators charge users a price that varies according to real-time energy generation cost. Since usually generation cost increases as the amount of generated energy increases, users may want to shift their load demands from peak hours to other times. Therefore, RTP can reduce the peak-hour load demand in the power system, which in turn lowers the requirement on system generation capacity. It can also reduce users' electricity bills by encouraging them to consume more power during hours with lower electricity prices. Another feature of the smart grid is *distributed generation (DG)*, where users install and take advantage of renewable generation resources (such as solar panels and wind turbines), and energy storage devices (e.g., batteries). In DG, users determine whether to immediately consume their own (generated or stored) energy, store it, or sell it to the grid. Thus, DG can help reduce the energy loss due to transmission inefficiencies, alleviate congestion during peak hours, reduce the system's carbon footprints, and lower users' electricity bills.

Due to unpredictable realtime prices and distributed energy resources, the smart grid poses great challenges for energy management (or load scheduling) with RTP and DG. Most previous studies focus on obtaining load schedules for customers in day-ahead scenarios based on the their load requirement. In particular, Goudarzi *et al.* [5] propose a mixed-integer optimization problem to find a load schedule that minimizes a customer's energy consumption cost plus an inconvenience function. Du *et al.* [6] present a two-step optimization algorithm to minimize a user's energy cost to run thermostatically controlled appliances. Gatsis and Giannakis [7] develop a day-ahead scheduling scheme considering imperfect information between the utility company and the customers due to packet loss. Mohsenian-Rad *et al.* [8] employ game theory to find an optimal daily load schedule for each user that minimizes the total energy generation cost. Shinwari *et al.* [9] design a water-filling based algorithm, which results in almost flat total power consumption of a neighborhood so as to minimize the changes in load demand per hour and reduce the utility company's operational costs. Salinas *et al.* [10] investigate a constrained multi-objective optimization problem (CMOP) to manage the energy consumption of a group of users. They develop two evolutionary algorithms to obtain the Pareto-front solutions and the ϵ-Pareto front solutions to the CMOP, respectively. Joe-Wong *et al.* [11] formulate a linear optimization problem to maximize the utility company's revenue. Note that all these studies require users to know exactly their load demands ahead of time, which may not be always predicted and can be uncertain. Besides, none of the above studies considers DG, energy storage management, or the possibility of users selling energy to the grid, which are essential and appealing features of the SG. In contrast, Neely *et al.* [12] develop an algorithm to minimize the long-term average expected cost of a utility company, which supplies power by a traditional power plant and a renewable energy resource. Individual user's load demand and energy storage devices are not considered. In [13], Urgaonkar *et al.* study a similar problem for a data center with an uninterruptible power supply that acts as an energy storage device. Guo *et al.* [14] propose an algorithm to minimize one user's long term expected energy cost considering a renewable energy resource and a battery. Note that essentially these works deal with one single load demand.

In this paper, we investigate the optimal energy management problem in the smart grid, taking into account customers' uncertain load demands, and distributed renewable energy resources and energy storage devices. Specifically, we consider an electric power distribution network consisting of a set of energy users, who have two-way real-time communications with a utility company. Each user has a renewable energy resource, an energy storage device, and a connection to the power grid, which collaboratively satisfy its load demand. The utility company provides energy to the users from both a traditional power plant (e.g., coal, gas) and a renewable energy resource (e.g., solar bank, wind farm). We model users' load demands and all renewable energy resources' as stochastic processes to account for their uncertainty. Besides, we consider that the system works in a time-slotted fashion. We aim to optimally schedule the usage of all the energy resources in the network and minimize the utility company's long-term time averaged expected total cost of supporting all users' load demands.

Moreover, we study two cases of users' load demands: first, users have delay intolerant (DI) load demands which need to be satisfied in the same time slot when they are requested, and second, users have both DI and delay tolerant (DT) load demands, the latter of which can tolerate being served within user-defined deadlines. In each case, we first formulate an optimization problem, which turns out to be a time-coupling problem. Previous approaches usually solve such problems based on Dynamic Programming [15], [16] and suffer from the "curse of dimensionality" problem [17]. They also require full statistical information of the random variables in the problem, which may be difficult to obtain in practice. Instead, we reformulate the problem using Lyapunov optimization theory for event-driven queueing systems [18]. We develop a dynamic energy management scheme that can dynamically solve the problem in each time slot based on the current system state only, i.e., without any information about the future or past system states, and hence is more efficient than previous approaches. With the results of our dynamic energy management scheme, we are then able to obtain both a lower and an upper bound on the optimal result of the original optimization problem. Furthermore, in the case of both DI and DT load demands, we also show that DT load demands are guaranteed to be served within user-defined deadlines. Extensive simulations have been conducted to evaluate the performance of the proposed dynamic energy management scheme. Results show that the proposed scheme can lead to tight lower and upper bounds on the optimal result, and can significantly reduce the utility company's cost.

The rest of the paper is organized as follows. Section II introduces system models considered in this study. We study dynamic energy management with DI load demands in Section III and with both DI and DT loads in Section IV. Simulations are conducted in Section V. We finally conclude this paper in Section VI.

II. System Model

In this section we describe the considered smart grid network and our mathematical models for users' delay intolerant load demand, distributed renewable energy generation, distributed energy storage, load serving, and the utility company's energy generation cost. Note that we only introduce delay intolerant load demand model here. Delay tolerant load demand model will be discussed in Section IV.

A. Smart Grid Network

We consider an electric power distribution network consisting of a set of residential and business energy users, denoted by $\mathcal{I} = \{1, 2, \ldots, n\}$, who have two-way real-time communications with a utility company. Each user has a renewable energy resource, an energy storage device, and a connection to the power grid, which collaboratively satisfy its load demand. The utility company provides energy to the users from both a traditional power plant (e.g., coal, gas) and a renewable energy resource (e.g., solar bank, wind farm). It aims to optimally schedule the usage of all the energy resources in the network and minimize its total cost of supporting all users' load demands. Besides, we consider that the system works in a time-slotted fashion. Energy management decisions are made dynamically by the utility company in each time slot. In particular, in each time slot, users transmit their load requests along with other state variables to a control center deployed by the utility company. Based on the collected data, the control center computes a load servicing schedule and transmits to each user his/her corresponding actions needed to be executed in the current time slot. Each user then follows the instructions and updates some of his/her state variables.

B. Delay Intolerant Load Demand Model

DI load demands are very common in our daily life, such as lighting and using electronic devices, and need to be satisfied in the same time slot. Denote user i's delay intolerant (DI) load demand in time slot t by $l_i(t)$. We assume $\{l_i(t)\}_{t=0}^{\infty}$ is an independent and identically distributed (i.i.d.) non-negative stochastic process, which is deterministically bounded, i.e., $0 \leq l_i(t) \leq l_i^{\max}$.

C. Distributed Renewable Energy Generation

Each user is equipped with a renewable energy resource, which can be a set of solar panels or a wind turbine. The output of a renewable energy resource is dynamic and difficult to predict because it depends on meteorological conditions. In this work, we assume that the output of user i's renewable energy resource, denoted by $e_i(t)$, is an i.i.d. stochastic process and satisfies $0 \leq e_i(t) \leq e_i^{\max}$, where $e_i^{\max}$ is the maximum energy output of user i's renewable energy resource and a constant.

In addition to serving user i's load, $e_i(t)$ can be used to charge the user's energy storage device, or sold to the power grid. In particular, we have

$$e_i(t) = r_i^l(t) + r_i^g(t) + c_i^r(t) \tag{1}$$

where $r_i^l(t)$ is the energy used to satisfy user i's load demand $l_i(t)$, $r_i^g(t)$ is the energy sold to the grid, and $c_i^r(t)$ is the energy used to charge user i's energy storage device.

D. Distributed Energy Storage

Each user i has an energy storage device which can store some energy that can be used at a later time. Since the energy storage device acts as an energy buffer, we can model its energy level as a queue, i.e.,

$$B_i(t+1) = B_i(t) + C_i(t) - D_i(t). \tag{2}$$

In particular, $C_i(t)$ is the energy charging the energy storage device, i.e.,

$$C_i(t) = c_i^g(t) + c_i^r(t) \tag{3}$$

where $c_i^g(t)$ and $c_i^r(t)$ are the energy drawn from the grid and from the renewable energy resource, respectively. $D_i(t)$ is the energy discharged from the energy storage device, i.e.,

$$D_i(t) = d_i^g(t) + d_i^l(t) \tag{4}$$

where $d_i^g(t)$ is the energy sold to the grid, and $d_i^l(t)$ is the energy serving user i's DI load demand.

Notice that it is more efficient to serve user i's load demand $l_i(t)$ by directly using energy from the grid or from the renewable energy resource, than by first charging the energy storage device and then discharging it. Thus, we have the following two constraints

$$\mathbf{1}_{d_i^l(t)>0} + \mathbf{1}_{c_i^r(t)>0} \leq 1 \tag{5}$$
$$\mathbf{1}_{d_i^l(t)>0} + \mathbf{1}_{c_i^g(t)>0} \leq 1 \tag{6}$$

where the indicator function $\mathbf{1}_A$ is equal to 1 when the event A is true, and zero otherwise.

On the other hand, it is more efficient to sell energy to the grid by directly selling the output of the renewable energy resource, than by first charging the energy storage device and then discharging it. Thus, we have

$$\mathbf{1}_{d_i^g(t)>0} + \mathbf{1}_{c_i^r(t)>0} \leq 1 \tag{7}$$

Similarly, discharging the energy storage device to sell energy to the grid and charging it by drawing energy from the grid cannot take place at the same time, i.e.,

$$\mathbf{1}_{d_i^g(t)>0} + \mathbf{1}_{c_i^g(t)>0} \leq 1 \tag{8}$$

The above constraints (5)–(8) will always hold when the following one holds:

$$\mathbf{1}_{C_i(t)>0} + \mathbf{1}_{D_i(t)>0} \leq 1 \tag{9}$$

Besides, denote by $B_i^{\max}$ the maximum amount of energy that can be stored by user i's energy storage device. Then, we need

$$0 \leq B_i(t) \leq B_i^{\max}. \quad (10)$$

Denote by $C_i^{\max}$ the maximum amount of energy that user i's energy storage device can be charged with during a single time slot, and $D_i^{\max}$ the maximum amount of energy that can be discharged from user i's energy storage device during a single time slot. Thus, we have

$$C_i(t) \leq \min\left[C_i^{\max}, B_i^{\max} - B_i(t)\right] \quad (11)$$
$$D_i(t) \leq \min\left[D_i^{\max}, B_i(t)\right]. \quad (12)$$

From (11) and (12), we get $C_i(t) + D_i(t) \leq B_i^{\max} - B_i(t) + B_i(t) = B_i^{\max}$, which should hold for any $C_i(t)$ and $D_i(t)$ that satisfy (11) and (12). Since $C_i(t) \leq C_i^{\max}$ and $D_i(t) \leq D_i^{\max}$, we also have the following constraint:

$$C_i^{\max} + D_i^{\max} \leq B_i^{\max}. \quad (13)$$

E. Load Serving

The utility company needs to supply enough energy to the grid to satisfy all users' load demands. The amount of energy supplied by the utility company in time slot t, denoted by $P(t)$, can be calculated as

$$P(t) = \sum_{i \in \mathcal{I}} \left(l_i(t) + c_i^g(t) - r_i^l(t) - r_i^g(t) - d_i^g(t) - d_i^l(t)\right). \quad (14)$$

User i's load demand is satisfied by the energy from the power grid, its local renewable energy resource, and its own energy storage device. Particularly, we have

$$l_i(t) = g_i^l(t) + r_i^l(t) + d_i^l(t) \quad (15)$$

where $g_i^l(t)$ is the amount of energy drawn from the power grid to satisfy user i's load demand in time slot t. Note that user i's connection to the power grid can only be in one of three states: drawing energy from the grid, providing energy to the grid, and idle, i.e., cannot draw and provide energy at the same time. Therefore, we get

$$\mathbf{1}_{g_i^l(t)+c_i^g(t)>0} + \mathbf{1}_{d_i^g(t)+r_i^g(t)>0} \leq 1. \quad (16)$$

In addition, the total amount of energy that user i draws from the power grid in time slot t, denoted by $G_i(t)$, satisfies

$$0 \leq G_i(t) = g_i^l(t) + c_i^g(t) \leq G_i^{\max} \quad (17)$$

where $G_i^{\max}$ is a constant determined by the physical characteristics of user i's connection to the grid. Similarly, the total amount of energy that user i provides to the power grid in time slot t, denoted by $M_i(t)$, satisfies

$$0 \leq M_i(t) = r_i^g(t) + d_i^g(t) \leq M_i^{\max} \quad (18)$$

where $M_i^{\max}$ is also a constant.

F. Energy Generation Cost

As mentioned before, the utility company provides energy to the users from both a traditional power plant and a renewable energy resource. Assume the output of the utility company's renewable energy resource, denoted by $R(t)$, is an i.i.d. non-negative stochastic process. The cost of generating such renewable energy is considered to be negligible. Thus, the utility company will first use renewable energy and then traditional energy to satisfy users' load demands. The amount of traditional energy the utility company needs in time slot t, denoted by $N(t)$, is

$$N(t) = P(t) - R(t) \quad (19)$$

If $R(t) > P(t)$, then the utility company is able to sell the excess power to other utility companies.

Consequently, a utility company's energy generation cost can be calculated as

$$U(t) = f(N(t)) \quad (20)$$

where $f(N(t))$ is a non-decreasing and convex function[1].

III. Dynamic Energy Management With Delay Intolerant Load Demands

In this section, we study the dynamic energy management for the smart grid when users have delay intolerant (DI) load demands.

A. Problem Formulation

Let $\mathbf{H}(t) = \{H_1(t), H_2(t), \ldots, H_n(t)\}$ be the vector of decision variables in the system, where $H_i(t) = \{g_i^l(t), d_i^g(t), d_i^l(t), c_i^r(t), c_i^g(t), r_i^g(t), r_i^l(t)\}$. We also denote the system state by a vector of random variables, i.e., $\mathbf{S}(t) = \{S_1(t), S_2(t), \ldots, S_n(t), R(t)\}$ where $S_i(t) = \{l_i(t), e_i(t)\}$. Thus, the utility company's objective is to design a dynamic energy management algorithm, which can optimally control the decision vectors $\mathbf{H}(t)$ $(t \geq 0)$ to minimize the following long-term time averaged expected total cost, i.e.,

$$\bar{U} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{U(t)\}, \quad (21)$$

under uncertain system states $\mathbf{S}(t)$ $(t \geq 0)$[2]. We call this problem $\mathbf{P1}$ and formally formulate it as follows:

$$\mathbf{P1} : \textbf{Minimize} \quad \bar{U}$$

We denote the optimal result, i.e., the minimum of the objective function, of $\mathbf{P1}$ by $\mathbf{P1}^*$. We can see that $\mathbf{P1}$ is a time-coupling optimization problem due to constraints (2), (10)–(12). Previous approaches usually solve such problems based on Dynamic Programming and suffer from the "curse of dimensionality" problem [17]. They also require detailed statistical information of the random variables in the problem, which may be

[1]Note that our analysis herein still holds if we assume a concave cost function f. In that case, our objective function can be set to $\max\{-f(N(t))\}$.

[2]Note that we use $\bar{x}$ to denote the long-term time averaged expected value of a stochastic process $x(t)$ in this study.

difficult to obtain in practice. Next, we reformulate this problem using Lyapunov optimization theory for queueing systems [18] so that it can be solved in each time slot based on the current system state only.

B. Dynamic Energy Management Using Lyapunov Optimization

In order to better control users' energy storage devices, we define a shifted energy level $X_i(t)$ for user i's energy storage device in time slot t as follows:

$$X_i(t) = B_i(t) - V\beta^{\max} - D_i^{\max}$$

where $\beta^{\max}$ is the maximum first-order derivative of $U(t)$ with respect to $N(t)$, and V is a positive constant to be defined later. We also denote by $\beta^{\min}$ the minimum first-order derivative of $U(t)$ with respect to $N(t)$.

Thus, according to (2), $X_i(t)$ is updated by the following queueing rule:

$$X_i(t+1) = X_i(t) + C_i(t) - D_i(t). \tag{22}$$

Consequently, we can define a Lyapunov function [18] as

$$L(\mathbf{X}(t)) = \frac{1}{2}\sum_{i\in\mathcal{I}}(X_i(t))^2.$$

where $\mathbf{X}(t) = \{X_1(t), \ldots, X_n(t)\}$. This function represents a scalar measure of stored energy in the system. $L(\mathbf{X}(t))$ being small implies that all stored energy levels are low, while $L(\mathbf{X}(t))$ being large implies that at least one stored energy level is high. Besides, the one-slot conditional Lyapunov drift can be defined as

$$\Delta(\mathbf{X}(t)) = \mathbb{E}\{L(\mathbf{X}(t+1)) - L(\mathbf{X}(t))\,|\,\mathbf{X}(t)\} \tag{23}$$

Since our objective is to minimize the long-term time averaged expected total cost of the utility company, instead of taking a control action to minimize $\Delta(\mathbf{X}(t))$, we minimize the following drift-plus-penalty function:

$$\Delta(\mathbf{X}(t)) + V\mathbb{E}\{U(t)\,|\,\mathbf{X}(t)\}.$$

We can have the following lemma.

Lemma 1: Given $\Delta(\mathbf{X}(t))$ defined in (23), we have

$$\Delta(\mathbf{X}(\mathbf{t})) + V\mathbb{E}\{U(t)\,|\,\mathbf{X}(t)\} \le A + V\mathbb{E}\{U(t)\,|\,\mathbf{X}(t)\} + \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{C_i(t) - D_i(t)\,|\,\mathbf{X}(t)\} \tag{24}$$

where A is a constant, i.e.,

$$A = \sum_{i\in\mathcal{I}}\left(\frac{\max\left[(C_i^{\max})^2, (D_i^{\max})^2\right]}{2}\right).$$

Proof: Squaring both sides of (22), we get

$$\begin{aligned}\frac{X_i^2(t+1) - X_i^2(t)}{2} &= \frac{(C_i(t) - D_i(t))^2}{2} + X_i(t)(C_i(t) - D_i(t))\\ &\le \frac{\max\left[(C_i^{\max})^2, (D_i^{\max})^2\right]}{2} + X_i(t)(C_i(t) - D_i(t)).\end{aligned}$$

Thus, we can obtain that

$$\begin{aligned}&\Delta(\mathbf{X}(\mathbf{t})) + V\mathbb{E}\{U(t)\,|\,\mathbf{X}(t)\}\\ &\le V\mathbb{E}\{U(t)\,|\,\mathbf{X}(t)\} + \mathbb{E}\left\{\sum_{i\in\mathcal{I}}\left(\frac{\max\left[(C_i^{\max})^2, (D_i^{\max})^2\right]}{2}\right.\right.\\ &\qquad \left.\left. + X_i(t)(C_i(t) - D_i(t))\right)\,\middle|\,\mathbf{X}(t)\right\},\end{aligned}$$

and (24) directly follows. ■

Our objective is to minimize the right-hand side of (24) in each time slot t given the current stored energy levels $\mathbf{X}(t)$ and system state $\mathbf{S}(t)$. Since A is a constant, we aim to minimize $VU(t) + \sum_{i\in\mathcal{I}} X_i(t)(C_i(t) - D_i(t))$. Moreover, recall that in $\mathbf{P1}$, constraints (2), (10)–(12) couple the energy levels of users' energy storage devices among all the time slots. We can break this coupling by leaving (2), (10) out, and relaxing (11), (12) into two constraints as follows:

$$0 \le C_i(t) \le C_i^{\max} \tag{25}$$

$$0 \le D_i(t) \le D_i^{\max}. \tag{26}$$

Therefore, we can formulate a relaxed optimization problem called $\mathbf{P2}$ in the following:

$$\begin{aligned}\mathbf{P2}: \quad &\textbf{Minimize} \quad VU(t) + \sum_{i\in\mathcal{I}} X_i(t)(C_i(t) - D_i(t))\\ &\textbf{s.t.} \quad (1), (3), (4), (9), (13)\text{–}(20), (25), (26)\end{aligned}$$

Our dynamic energy management is carried out as follows. The utility company solves the problem $\mathbf{P2}$ in each time slot t given $\mathbf{X}(t)$ and $\mathbf{S}(t)$ collected from the users. It then sends the obtained control decisions to the users, who follow the instructions and update their stored energy levels $\mathbf{X}(t)$ according to (22) and (2). We denote the corresponding long-term time averaged expected total cost, i.e., $\bar{U}$, by $\mathbf{P2}^*$.

Theorem 1: Define the maximum value of V as

$$V^{\max} = \min_{i\in\mathcal{I}} \frac{B_i^{\max} - C_i^{\max} - D_i^{\max}}{\beta^{\max} - \beta^{\min}}.$$

For $0 \le B_i(0) \le B_i^{\max}$ for all $i \in I$ and any $0 \le V \le V^{\max}$, our dynamic energy management scheme has the following properties:

a) An arbitrary user i's stored energy level $B_i(t)$ satisfies the constraint (10), i.e., $0 \le B_i(t) \le B_i^{\max}$, for all $t \ge 0$.

b) The obtained control decisions are feasible solutions to $\mathbf{P1}$.

c) $\mathbf{P2}^* - A/V \le \mathbf{P1}^* \le \mathbf{P2}^*$.

Proof: a) We prove a) by induction. Particularly, assume that for an arbitrary user i, (10) holds in time slot t. Then, we consider the following cases to prove that (10) also holds in time slot $t+1$.

First, $0 \leq B_i(t) < D_i^{\max}$. Recall that $C_i(t) = c_i^g(t) + c_i^r(t)$. In this case, the partial derivative of the objective function of $\mathbf{P2}$, denoted by $P2(t)$, with respect to $c_i^g(t)$, is

$$\begin{aligned}\frac{\partial P2(t)}{\partial c_i^g(t)} &= V\frac{\partial U(t)}{\partial c_i^g(t)} + X_i(t)\\ &\leq V\beta^{\max} + B_i(t) - V\beta^{\max} - D_i^{\max}\\ &< 0.\end{aligned}$$

Similarly, we can have

$$\frac{\partial P2(t)}{\partial c_i^r(t)} = X_i(t) < -V\beta^{\max} < 0.$$

Thus, by solving $\mathbf{P2}$, i.e., minimizing $P2(t)$, our energy management scheme leads to control decisions that satisfy $C_i(t) = c_i^r(t)+c_i^g(t) = C_i^{\max}$. Due to constraint (9), we have $D_i(t) = 0$. Therefore, according to (2), we get $B_i(t+1) = B_i(t) + C_i^{\max}$ and hence

$$0 \leq B_i(t+1) \leq D_i^{\max} + C_i^{\max} \leq B_i^{\max}$$

due to constraint (13).

Second, $D_i^{\max} \leq B_i(t) \leq V(\beta^{\max} - \beta^{\min}) + D_i^{\max}$. Since

$$V \leq V^{\max} \leq \frac{B_i^{\max} - C_i^{\max} - D_i^{\max}}{\beta^{\max} - \beta^{\min}},$$

we have $B_i(t) \leq B_i^{\max} - C_i^{\max}$. Thus, according to (2), we can obtain

$$B_i(t+1) \leq B_i^{\max} - C_i^{\max} + C_i(t) - D_i(t) \leq B_i^{\max}$$
$$\text{and}$$
$$B_i(t+1) \geq D_i^{\max} + C_i(t) - D_i(t) \geq 0.$$

Third, $V(\beta^{\max} - \beta^{\min}) + D^{\max} < B_i(t) \leq B_i^{\max}$. Note that $V \leq (B_i^{\max} - C_i^{\max} - D_i^{\max})/(\beta^{\max} - \beta^{\min})$, and hence $V(\beta^{\max}-\beta^{\min})+D^{\max} \leq B_i^{\max}-C_i^{\max} < B_i^{\max}$. The partial derivative of the objective function of $\mathbf{P2}$ with respect to $d_i^g(t)$ is

$$\begin{aligned}\frac{\partial P2(t)}{\partial d_i^g(t)} &= -V\frac{\partial U(t)}{\partial d_i^g(t)} - X_i(t)\\ &\leq -V\beta^{\min} - B_i(t) + V\beta^{\max} + D_i^{\max}\\ &< 0.\end{aligned}$$

Similarly, we can also get that $\partial P2(t)/\partial d_i^l(t) < 0$. Thus, our energy management scheme minimizing $P2(t)$ results in control decisions that satisfy $D_i(t) = d_i^g(t) + d_i^l(t) = D_i^{\max}$. Due to constraint (9), we have $C_i(t) = 0$. Thus, according to (2), we get $B_i(t+1) = B_i(t) - D_i^{\max}$ and hence

$$0 \leq B_i(t+1) \leq B_i^{\max} - D_i^{\max} \leq B_i^{\max}.$$

Therefore, we can see that (10) holds for all $t \geq 0$.

b) We have known from a) that constraint (10) holds. Besides, according to (2), we have $C_i(t) = B_i(t+1) - B_i(t) + D_i(t) \leq B_i^{\max} - B_i(t) + D_i(t)$. Due to constraint (9), we have that $D_i(t) = 0$ when $C_i(t) > 0$. Thus, we get $C_i(t) \leq B_i^{\max} - B_i(t)$. Furthermore, we have $B_i(t+1) = B_i(t) + C_i(t) - D_i(t) \geq 0$, which leads to $D_i(t) \leq B_i(t) + C_i(t)$. Similarly, since $C_i(t) = 0$ when $D_i(t) > 0$, we get $D_i(t) \leq B_i(t)$. Therefore, both (11) and (12) hold as well. In addition, our dynamic energy management scheme updates the stored energy levels $\mathbf{X(t)}$ according to (22), which means (22) holds too. As a result, the control decisions obtained by our dynamic energy management scheme satisfy all the constraints of $\mathbf{P1}$, and hence are feasible solutions to $\mathbf{P1}$.

c) Denote by $\widehat{C_i}(t)$, $\widehat{D_i}(t)$, and $\widehat{U}(t)$ the results obtained by our dynamic energy management scheme in time slot t, i.e., based on the optimal solution to $\mathbf{P2}$. We also denote by $C_i^*(t)$, $D_i^*(t)$, and $U^*(t)$ the results that we get for time slot t based on the optimal solution to $\mathbf{P1}$. Thus, from Lemma 1, we can have

$$\begin{aligned}&\Delta(\mathbf{X(t)}) + V\mathbb{E}\{\widehat{U}(t)\,|\,\mathbf{X}(t)\}\\ &\quad\leq A + V\mathbb{E}\{\widehat{U}(t)\,|\,\mathbf{X}(t)\}\\ &\qquad+ \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{\widehat{C_i}(t) - \widehat{D_i}(t)\,|\,\mathbf{X}(t)\}\\ &\quad\leq A + V\mathbb{E}\{U^*(t)\,|\,\mathbf{X}(t)\}\\ &\qquad+ \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{C_i^*(t) - D_i^*(t)\,|\,\mathbf{X}(t)\}\\ &\quad= A + V\mathbb{E}\{U^*(t)\} + \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{C_i^*(t) - D_i^*(t)\}\end{aligned}$$

Note that the last step is due to the fact that the optimal solutions to $\mathbf{P1}$ are obtained independent of the current stored energy levels.

Besides, since the system state $\mathbf{S}(t)$ is i.i.d., it follows that $C_i^*(t)$ and $D_i^*(t)$ are also i.i.d. stochastic processes. Recall the strong law of large numbers: If $\{a(t)\}_{t=0}^{\infty}$ are i.i.d. random variables, we have $\Pr((1/T)\lim_{T\to\infty}\sum_{t=0}^{T-1} a(t) = \mathbb{E}\{a(t)\}) = 1$ almost surely. Consequently, we get

$$\begin{aligned}&\mathbb{E}\{L(\mathbf{X}(t+1)) - L(\mathbf{X}(t))\,|\,\mathbf{X}(t)\} + V\mathbb{E}\{\widehat{U}(t)\,|\,\mathbf{X(t)}\}\\ &\leq A + V\mathbb{E}\{U^*(t)\} + \sum_{i\in\mathcal{I}} X_i(t)\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}(C_i^*(t) - D_i^*(t))\end{aligned}$$

Taking expectation of the above inequality, we get

$$\begin{aligned}&\mathbb{E}\{L(\mathbf{X}(t+1))\} - \mathbb{E}\{L(\mathbf{X}(t))\} + V\mathbb{E}\{\widehat{U}(t)\}\\ &\quad\leq A + V\mathbb{E}\{U^*(t)\}\\ &\qquad+ \sum_{i\in\mathcal{I}} \mathbb{E}\{X_i(t)\}\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{C_i^*(t) - D_i^*(t)\}\\ &\quad= A + V\mathbb{E}\{U^*(t)\} + \sum_{i\in\mathcal{I}} \mathbb{E}\{X_i(t)\}(\overline{C_i^*} - \overline{D_i^*}).\end{aligned}$$

In addition, summing (2) over all the time slots $t \in \{0, 1, 2, \ldots, T-1\}$ and taking expectation on both sides, we have

$$\mathbb{E}\{B_i(T)\} - \mathbb{E}\{B_i(0)\} = \sum_{t=0}^{T-1}\mathbb{E}\{C_i^*(t) - D_i^*(t)\} \tag{27}$$

Dividing the above equation by T and taking limits as $T \to \infty$, we get $\overline{C_i^*} - \overline{D_i^*} = 0$. Therefore, we can obtain

$$\begin{aligned}\mathbb{E}\{L(\mathbf{X}(t+1))\} - \mathbb{E}\{L(\mathbf{X}(t))\} + V\mathbb{E}\{\widehat{U}(t)\}&\\ \leq A + V\mathbb{E}\{U^*(t)\}.&\end{aligned}$$

Summing the above inequality over all the time slots $t \in \{0, 1, 2, \ldots, T-1\}$, we get

$$\sum_{t=0}^{T-1} V\mathbb{E}\{\widehat{U}(t)\} \leq AT + V\sum_{t=0}^{T-1}\mathbb{E}\{U^*(t)\} - \mathbb{E}\{L(\mathbf{X}(T))\} + \mathbb{E}\{L(\mathbf{X}(0))\}$$

Since $0 \leq B_i(t) \leq B_i^{\max}$ for all $t \geq 0$, $X_i(t)$ is finite in all time slots as well. Then, dividing both sides of the above inequality by VT and taking limits as $T \to \infty$, we can obtain

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{\widehat{U}(t)\} \leq \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{U^*(t)\} + \frac{A}{V},$$

which means $\mathbf{P1}^* \geq \mathbf{P2}^* - A/V$.

Besides, as shown in b), the optimal solutions to $\mathbf{P2}$ are also a feasible solution to $\mathbf{P1}$. Thus, the value of the objective function of $\mathbf{P1}$ calculated based on the optimal solution to $\mathbf{P2}$, i.e., $\mathbf{P2}^*$, is an upper bound on $\mathbf{P1}^*$, i.e., $\mathbf{P1}^* \leq \mathbf{P2}^*$.

We have now finished the proof. ■

IV. Dynamic Energy Management With Mixed Load Demands

In this section, we extend the basic system model described in Section II to the case that users have both delay intolerant (DI) and delay tolerant (DT) load demands. In particular, the same as before, DI load demands need to be satisfied in the same time slots when they are requested without any delay. In contrast, DT load demands just need to be served before some user-defined deadlines. Examples for DT load demands are washer/dryer machines, dishwashers, etc.

A. Mixed Load Demand Model

Consider that an arbitrary user i has both DI and DT load demands. DI load demands are modeled in the same way as described in Section II-B. We denote user i's DT load demand in time slot t by $T_i(t)$. We also assume that $\{T_i(t)\}_{t=0}^{\infty}$ is an i.i.d. non-negative stochastic process, and $0 \leq T_i(t) \leq T_i^{\max}$ for all $t \geq 0$. Besides, we assume $T_i^{\max} \leq G_i^{\max}$. It means that the DT load demand that a user can have in one slot is no larger than the maximum amount of energy it can draw from the power grid, which is reasonable.

User i's DT demand is placed in a local queue $Q_i(t)$, which is updated as follows:

$$Q_i(t+1) = \max[Q_i(t) - y_i(t), 0] + T_i(t) \tag{28}$$

where $y_i(t) = d_i^q(t) + g_i^q(t) + r_i^q(t)$ is the amount of service received by the queue. Particularly, $d_i^q(t)$, $g_i^q(t)$, and $r_i^q(t)$ are the energy drawn from user i's energy storage device, the power grid, and user i's renewable energy resource in time slot t to support user i's DT demand, respectively.

Due to the introduction of DT load demands, constraint (1) changes into:

$$e_i(t) = r_i^l(t) + r_i^q(t) + r_i^g(t) + c_i^r(t). \tag{29}$$

$C_i(t)$ remains the same, while $D_i(t)$ changes from (4) into:

$$D_i(t) = d_i^g(t) + d_i^l(t) + d_i^q(t). \tag{30}$$

$M_i(t)$ remains the same, while $G_i(t)$ changes from (17) into:

$$0 \leq G_i(t) = g_i^l(t) + g_i^q(t) + c_i^g(t) \leq G_i^{\max}, \tag{31}$$

and (16) changes into

$$\mathbf{1}_{g_i^l(t)+c_i^g(t)+g_i^q(t)>0} + \mathbf{1}_{d_i^g(t)+r_i^g(t)>0} \leq 1. \tag{32}$$

Besides, the amount of energy supplied by the utility company in time slot t changes from (14) into

$$P(t) = \sum_{i\in\mathcal{I}}\left(l_i(t) + g_i^q(t) + c_i^g(t) - r_i^l(t) - r_i^g(t) - d_i^g(t) - d_i^l(t)\right). \tag{33}$$

B. Problem Formulation With Mixed Load Demand Model

Let $\mathbf{H}(t) = \{H_1(t), H_2(t), \ldots, H_n(t)\}$ be the vector of decision variables in the system, where $H_i(t) = \{g_i^l(t), g_i^q(t), d_i^g(t), d_i^l(t), d_i^q(t), c_i^r(t), c_i^g(t), r_i^g(t), r_i^l(t), r_i^q(t)\}$. We also denote the system state by a vector of random variables, i.e., $\mathbf{S}(t) = \{S_1(t), S_2(t), \ldots, S_n(t), R(t)\}$ where $S_i(t) = \{l_i(t), T_i(t), e_i(t)\}$. Thus, the dynamic energy management problem with mixed load demand model, which we call $\mathbf{P3}$, can be formulated as follows:

$$\begin{aligned}
\mathbf{P3}:\ & \mathbf{Minimize}\quad \bar{U}\\
\mathbf{s.t.}\ & \text{DT loads are served before user-defined deadlines}\\
& \text{Constraints}: (2), (3), (9)\text{–}(13), (15), (18)\text{–}(20),\\
& (29)\text{–}(33), \quad \forall t \geq 0
\end{aligned}$$

We denote the optimal result, i.e., the minimum of the objective function, of $\mathbf{P3}$ by $\mathbf{P3}^*$. We notice that $\mathbf{P3}$ is also a time-coupling optimization problem, which is prohibitively difficult to solve as explained in Section III-A. Similarly, in what follows we reformulate this problem based on Lyapunov optimization theory such that it can be solved based on current system state only.

C. Delay Aware Virtual Queue

In order to characterize the delay in serving users' DT load demand, we define a delay-aware virtual queue $Z_i(t)$ for each user i, whose queueing function is as follows:

$$Z_i(t+1) = \max[Z_i(t) - y_i(t), 0] + \epsilon_i \mathbf{1}_{Q_i(t)>0}. \tag{34}$$

In particular, $Z_i(t)$ has the same serving rate as $Q_i(t)$, but a different arrival rate. ϵ_i is a constant related to user-defined service deadline, which will be specified in Lemma 2. We also assume that $\epsilon_i \leq G_i^{\max}$, i.e., the arriving rate is no larger than the maximum amount of energy user i can draw from the power grid. We have the following lemma.

Lemma 2: Assume that the queues $\mathbf{Q}(t)$ and $\mathbf{Z}(t)$ are controlled in such a way that $Q_i(t) < Q_i^{\max}$ and $Z_i(t) < Z_i^{\max}$ for all $t \geq 0$ and $i \in \mathcal{I}$, where $Z_i^{\max}$ and $Q_i^{\max}$ are deterministic positive constants. Then, an arbitrary user i's DT load demand $T_i(t)$ can be served within a maximum delay of

$$\mu_i^{\max} = \left\lceil \frac{Q_i^{\max} + Z_i^{\max}}{\epsilon_i} \right\rceil. \tag{35}$$

Proof: In what follows, we prove (35) by contradiction.

Assume that the delay in serving an arbitrary user i's DT demand is larger than $\mu_i^{\max}$. Suppose $T_i(t) > 0$ in time slot t. Thus, we have $Q(t+1) > 0$ according to (28), and $Q(\tau) > 0$ for $t+1 \leq \tau \leq t + \mu_i^{\max}$. Referring to (34), we get

$$Z_i(\tau+1) \geq Z_i(\tau) - y_i(\tau) + \epsilon_i$$

for $t+1 \leq \tau \leq t+\mu_i^{\max}$. Summing over the time slots from $t+1$ to $t+\mu_i^{\max}$ yields

$$Z_i\left(t+\mu_i^{\max}+1\right) - Z_i(t+1) \geq \mu_i^{\max}\epsilon_i - \sum_{\tau=t+1}^{t+\mu_i^{\max}} y_i(\tau)$$

Since $Z(t+\mu_i^{\max}+1) \leq Z_{\max}$ and $Z(t+1) \geq 0$, we can get

$$\sum_{t+1}^{t+\mu_i^{\max}} y_i(t) \geq \mu_i^{\max}\,\epsilon_i - Z_i^{\max} \tag{36}$$

Consider that the DT loads are served in a first in first out (FIFO) manner. Since $Q_i(t+1) < Q_i^{\max}$ and user i's DT demand $T_i(t)$ has not been served by $t+\mu_i^{\max}$, we have $\sum_{\tau=t+1}^{t+\mu_i^{\max}} y_i(t) < Q_i^{\max}$. Thus, from (36) we can get

$$Q_i^{\max} + Z_i^{\max} > \mu_i^{\max}\epsilon_i.$$

Since $\mu_i^{\max} = \lceil (Q_i^{\max} + Z_i^{\max})/(\epsilon_i) \rceil$, we have $Q_i^{\max} + Z_i^{\max} > Q_i^{\max} + Z_i^{\max}$, which is impossible. Thus, the assumption that the delay in serving an arbitrary user i's DT demand is larger than $\mu_i^{\max}$ is invalid, and Lemma 2 follows. ■

According to Lemma 2, each user i can set ϵ_i based on $Q_i^{\max}$ and $Z_i^{\max}$ to make sure that its DT load demand can be satisfied by a certain deadline. We will describe $Q_i^{\max}$ and $Z_i^{\max}$ in detail later. We are now ready to present our Lyapunov optimization based energy management scheme.

D. Dynamic Energy Management Based on Lyapunov Optimization

Notice that the queues that are maintained in the system can be denoted by a vector $\mathbf{\Theta}(t) = \{\mathbf{X}(t), \mathbf{Q}(t), \mathbf{Z}(t)\}$. Thus, we can define a Lyapunov function as

$$L(\mathbf{\Theta}(t)) = \frac{1}{2}\sum_{i\in\mathcal{I}}((X_i(t))^2 + (Q_i(t))^2 + (Z_i(t))^2),$$

and the one-slot conditional Lyapunov drift is

$$\Delta(\mathbf{\Theta}(t)) = \mathbb{E}\{L(\mathbf{\Theta}(t+1)) - L(\mathbf{\Theta}(t)) \,|\, \mathbf{\Theta}(t)\}. \tag{37}$$

Recall that $y_i(t) = d_i^q(t) + g_i^q(t) + r_i^q(t)$. Since $0 \leq d_i^q(t) \leq D_i^{\max}$, $0 \leq g_i^q(t) \leq G_i^{\max}$, and $0 \leq r_i^q(t) \leq e_i(t) \leq e_i^{\max}$, we have $0 \leq y_i(t) \leq D_i^{\max} + G_i^{\max} + e_i^{\max}$. We denote the upper bound on $y_i(t)$ as $y_i^{\max}$. Then, we can have the following lemma regarding the drift-plus-penalty function.

Lemma 3: Given $\Delta(\mathbf{\Theta}(t))$ defined in (37), we have

$$\begin{aligned}\Delta(\mathbf{\Theta}(t)) &+ V\mathbb{E}\{U(t) \,|\, \mathbf{\Theta}(\mathbf{t})\}\\ &\leq K + V\mathbb{E}\{U(t) \,|\, \mathbf{\Theta}(\mathbf{t})\}\\ &\quad + \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{C_i(t) - D_i(t) \,|\, \mathbf{\Theta}(\mathbf{t})\}\\ &\quad + \sum_{i\in\mathcal{I}} Q_i(t)\mathbb{E}\{T_i(t) - y_i(t) \,|\, \mathbf{\Theta}(\mathbf{t})\}\\ &\quad + \sum_{i\in\mathcal{I}} Z_i(t)\mathbb{E}\{\epsilon_i - y_i(t) \,|\, \mathbf{\Theta}(\mathbf{t})\}\end{aligned} \tag{38}$$

where K is a constant, i.e.,

$$K = \sum_{i\in\mathcal{I}}\left(\frac{\max\left[(C_i^{\max})^2, (D_i^{\max})^2\right]}{2} + \frac{(T_i^{\max})^2 + (y_i^{\max})^2}{2} + \frac{\epsilon_i^2 + (y_i^{\max})^2}{2}\right).$$

Proof: We have obtained in Lemma 1 that

$$\frac{X_i^2(t+1) - X_i^2(t)}{2} \leq \frac{\max\left[(C_i^{\max})^2, (D_i^{\max})^2\right]}{2} + X_i(t)(C_i(t) - D_i(t)). \tag{39}$$

Besides, note that $\forall x, y, z$ with $x \geq 0$, $0 \leq y \leq y_{\max}$, $0 \leq z \leq z_{\max}$, we have

$$\begin{aligned}(\max\{x-y, 0\} + z)^2 &\leq x^2 + y^2 + z^2 + 2x(z-y)\\ &\leq x^2 + y_{\max}^2 + z_{\max}^2 + 2x(z-y).\end{aligned}$$

Thus, squaring both sides of (28), we get

$$\frac{Q_i(t+1)^2 - Q_i(t)^2}{2} \leq \frac{(T_i^{\max})^2 + (y_i^{\max})^2}{2} + Q_i(t)\left(T_i(t) - y_i(t)\right) \tag{40}$$

Similarly, squaring both sides of (34), we have

$$\frac{Z_i(t+1)^2 - Z_i(t)^2}{2} \leq \frac{\epsilon_i^2 + (y_i^{\max})^2}{2} + Z_i(t)(\epsilon_i - y_i(t)). \tag{41}$$

Therefore, summing (39), (40), and (41) over all $i \in \mathcal{I}$, taking expectations conditioned on $\mathbf{\Theta}(t)$, and adding the cost function $V\mathbb{E}\{U(t) \,|\, \mathbf{\Theta}(t)\}$, we arrive at Lemma 3. ■

Similar to that in Section III-B, we aim to minimize the right-hand side of (38) in each time slots t based on current system state. Note that in (38) $T_i(t)$ is a constant given the current system state, and ϵ_i is a constant, too. Thus, removing the constants and relaxing the constraints (2), (10)–(12), we formulate a new problem **P4** as follows:

$$\mathbf{P4}: \quad \textbf{Minimize} \quad VU(t) + \sum_{i\in\mathcal{I}}(X_i(t)(C_i(t) - D_i(t)) - (Q_i(t) + Z_i(t))y_i(t))$$

$$\textbf{s.t.} \quad \text{Constraints}: (3), (9), (13), (15), (18)\text{–}(20), (25), (26), (29)\text{–}(33)$$

Our dynamic energy management scheme works as follows. The utility company solves the problem $\mathbf{P4}$ in each time slot t given $\mathbf{\Theta}(t)$ and $\mathbf{S}(t)$ collected from the users. It then sends the obtained control decisions to the users, who follow the instructions and update their queues $\mathbf{X(t)}$, $\mathbf{Q}(t)$, and $\mathbf{Z}(t)$ in the system according to (22) and (2), (28), and (34), respectively. We denote the corresponding long-term time averaged expected total cost, i.e., $\bar{U}$, by $\mathbf{P4}^*$.

Theorem 2: Define the maximum value of V as

$$V^{\max} = \min_{i \in \mathcal{I}} \frac{B_i^{\max} - C_i^{\max} - D_i^{\max} - N_i}{\beta^{\max}}.$$

where $N_i = (4B_i^{\max} - 4C_i^{\max} - 2D_i^{\max} + 3T_i^{\max} + 3\epsilon_i)/7$. Assume $B_i^{\max} \gg C_i^{\max} + D_i^{\max} + T_i^{\max} + \epsilon_i$. Suppose all DT load demand queues and virtual queues start with zero backlogs, i.e., $Q_i(0) = Z_i(0) = 0$ for all $i \in \mathcal{I}$, and all energy storage devices start with feasible energy levels, i.e., $0 \le B_i(0) \le B_i^{\max}$ for all $i \in \mathcal{I}$. Then, for any $0 \le V \le V^{\max}$, our dynamic energy management scheme has the following properties:

a) For an arbitrary user i, its queues $Q_i(t)$ and $Z_i(t)$ are deterministically upper bounded by constants $Q_i^{\max}$ and $Z_i^{\max}$, respectively, for all $t \ge 0$ where

$$Q_i^{\max} = \frac{2V\beta^{\max} + D_i^{\max}}{3} + T_i^{\max} \tag{42}$$

$$Z_i^{\max} = \frac{2V\beta^{\max} + D_i^{\max}}{3} + \epsilon_i \tag{43}$$

b) For an arbitrary user i, its stored energy level $B_i(t)$ satisfies (10), i.e., $0 \le B_i(t) \le B_i^{\max}$ for all $t \ge 0$.

c) For an arbitrary user i, its DT load demand can be served with a maximum delay of

$$\mu_i^{\max} = \left\lceil \frac{4V\beta^{\max} + 2D_i^{\max} + 3T_i^{\max} + 3\epsilon_i}{3\epsilon_i} \right\rceil. \tag{44}$$

d) The obtained control solutions are feasible solutions to $\mathbf{P3}$.

e) $\mathbf{P4}^* - K/V \le \mathbf{P3}^* \le \mathbf{P4}^*$.

Proof: Please see the Appendix for detailed proof. ■

V. Simulation Results

In this section, we evaluate the performance of our dynamic energy management scheme using practical renewable energy generation data. We study two cases: when users have DI load demands only and when users have both DI and DT load demands. In each case, we first obtain the lower and upper bounds on the optimal result. Then, we calculate our total energy generation cost and compare it with that of a simple energy management strategy. We implement our proposed dynamic energy management schemes on a general purpose PC with 64-bit Windows 7, 25 GB RAM, and a 2.26 GHz CPU. Using CPLEX, we solve optimization problems $\mathbf{P2}$ and $\mathbf{P4}$ for the two cases, respectively.

Some simulation settings are as follows. We consider 10 users using energy for a period of 10 days with 5-minute long time slots, i.e., 3000 time slots in total. Users' renewable energy generation capabilities are set based on the global horizontal irradiance data for Las Vegas area available at the Measurement and Instrumentation Data Center [19]. In particular, we assume the energy conversion efficiency is 15% and the maximum output is 200 W. Besides, the maximum charging and discharging limits on each user's energy storage device in a time slot, i.e., $C_i^{\max}$ and $D_i^{\max}$, are both set to 1.5 kWh. The maximum amount of energy that each user can draw from the power grid in a time slot, i.e., $G_i^{\max}$, is set to the maximum load request plus the maximum charging limit in a time slot. The maximum amount of energy that each user can sell to the grid, i.e., $M_i^{\max}$, is set to be the same as $G_i^{\max}$. In addition, we ignore the utility company's renewable energy resource and focus on the management of users' energy resources in this simulation. So the utility company's energy generation cost function is defined as $U(t) = aP^2(t) + bP(t) + c$, where $a = 0.75, b = 0.1$ and $c = 0$.

In the case that users have DI load demands only, we consider that each user's DI load demands are i.i.d. uniform random variables over the interval [1, 7] kWh. Fig. 1(a) shows the upper and lower bounds on the optimal result. Note that the upper bound is the time averaged expected total cost during the whole simulation period, obtained by our dynamic energy management scheme. The lower bound is the upper bound minus A/V as shown in Theorem 1. In our simulations, we set $V = V^{\max}$. Recall that according to Theorem 1, A is independent of $B^{\max}$ while $V^{\max}$ increases as $B^{\max}$ increases. Thus, the performance bounds get tighter as $B^{\max}$ increases as we can see in Fig. 1(a). In addition, we compare in Fig. 1(b) the total energy generation cost of our dynamic energy management scheme since $t = 0$ with that of a simple energy management strategy. In particular, the simple strategy satisfies users' DI load demands in the same time slot when they are requested. It does not consider users selling energy to the grid or using energy storage devices. We can observe noticeable savings using our scheme, which keep increasing as time goes by.

In the case that each user has both a DI and a DT load demands, we consider that each user's both load demands are i.i.d. uniform random variables over the interval [1, 3.5] kWh. We set all DT load demand deadlines to 168 hours (7 days), i.e., $\mu_i^{\max} = 2016$, and set ϵ_i according to (44) for each energy storage device size. We show the upper and lower bounds on the optimal result in Fig. 2(a), and find that the bounds get tighter as $B^{\max}$ increases. We also compare the total energy generation cost of our dynamic energy management scheme since $t = 0$ with that of a simple energy management strategy in Fig. 2(b). In particular, the simple strategy satisfies users' DI and DT load demands in the same time slot when they are requested. It does not consider users selling energy to the grid or using energy storage devices. We can observe noticeable savings using our scheme as well. Fig. 2(c) shows the time that it takes DT load demands to be satisfied when each user has an energy storage device with capability of $B^{\max} = 240$ kWh. We observe that all DT loads can be served within 15 hours, much earlier than the user-defined deadline. In Fig. 2(d), we present the energy level of a user's energy storage device which always remains within its physical limits as described in Theorem 2.

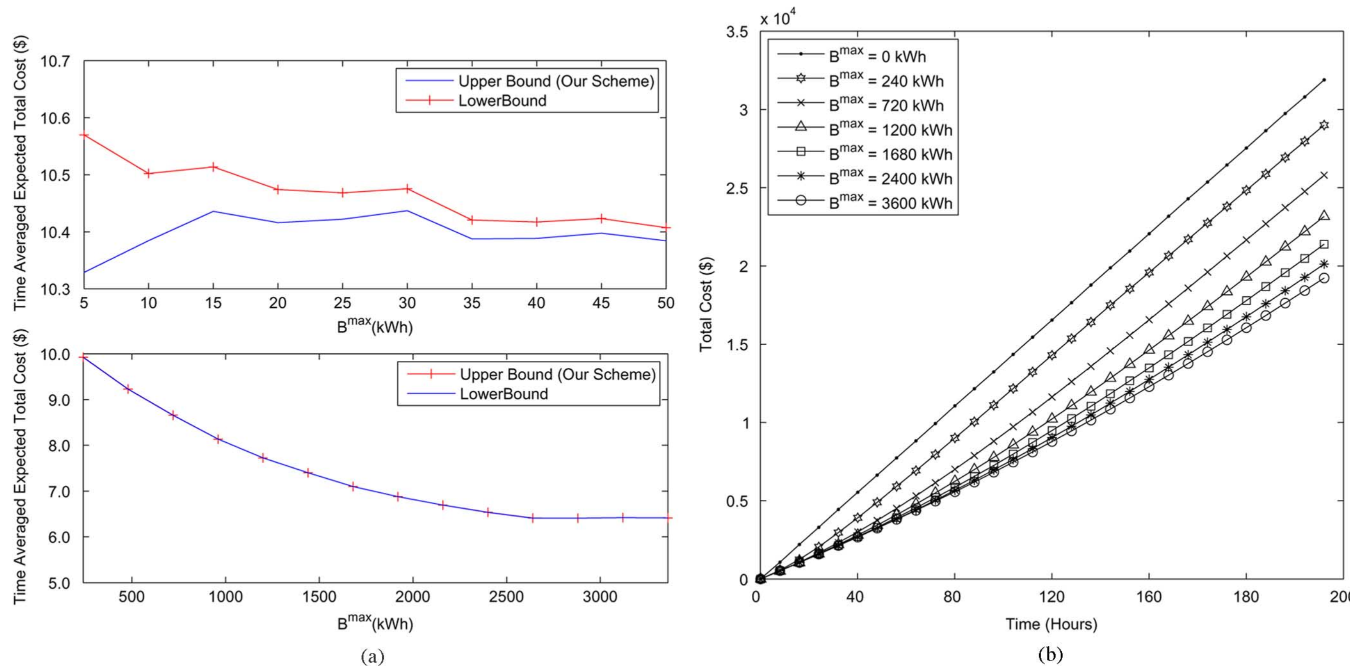

Fig. 1. The case of DI load demands. (a) Bounds on optimal time averaged expected total cost. (b) Total generation cost.

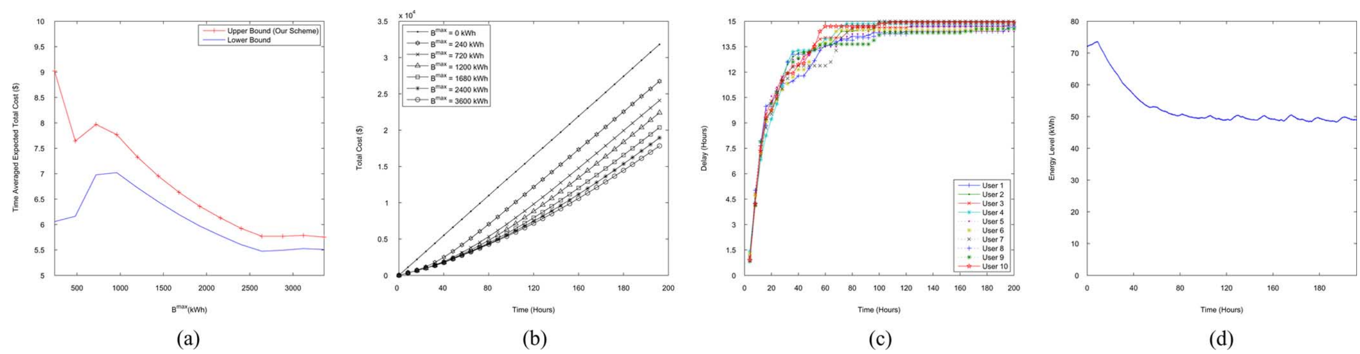

Fig. 2. The case of both DI and DT load demands. (a) Bounds on optimal time averaged expected total cost. (b) Total generation cost. (c) Service delays of DT load demands when $B^{\max} = 240$ kWh. (d) Energy storage device behavior when $B^{\max} = 240$ kWh.

Moreover, although in our dynamic energy management schemes, the optimization problems $\mathbf{P2}$ and $\mathbf{P4}$ need to be solved once every time slot, we find that on average they can be solved in about 0.3 seconds on the PC we use for our simulations. The computation time is very low and can be even lower on more powerful computers.

VI. Conclusion

In this paper, we have explored dynamic energy management in the smart grid, considering unpredictable load demands, and distributed uncertain renewable energy resources and energy storage devices. We have studied two kinds of user load demands: DI demands only, and both DI and DT demands. In particular, with the objective of minimizing the long-term time averaged expected total cost of supporting all users' load demands, we formulate an optimization problem, which is a time-coupling problem and prohibitively expensive to solve. Then, employing Lyapunov optimization theory, we reformulate the problem and develop a dynamic energy management scheme which can dynamically solve the problem in each time slot. The developed scheme result in both a lower and an upper bound on the optimal result of the original optimization problem. Furthermore, in the case of both DI and DT load demands, we show that DT load demands are guaranteed to be served within user-defined deadlines. Extensive simulation results are presented to validate the efficiency of the proposed scheme.

Appendix A
Proof of Theorem 2

a) We first prove (42) by induction. Obviously, (42) holds for $t = 0$. Assume that (42) holds in time slot t. In the following, we show that (42) also holds in time slot $t+1$.

First, $0 \le Q_i(t) \le (2V\beta^{\max} + D_i^{\max})/(3)$. Since $T_i(t) \le T_i^{\max}$, then according to (28), we have

$$Q_i(t+1) \le \max\left[Q_i(t) - y_i(t) + T_i^{\max}, T_i^{\max}\right].$$

Thus, we get $Q_i(t+1) \le Q_i(t) + T_i^{\max} \le (2V\beta^{\max} + D_i^{\max})/(3) + T_i^{\max}$.

Second, $(2V\beta^{\max} + D_i^{\max})/(3) < Q_i(t) \leq (2V\beta^{\max} + D_i^{\max})/(3) + T_i^{\max}$. In this case, the partial derivative of the objective function of **P4**, denoted by $P4(t)$, with respect to $g_i^q(t)$, is

$$\begin{aligned}\frac{\partial P4(t)}{\partial g_i^q(t)} &= V\frac{\partial U(t)}{\partial g_i^q(t)} - (Q_i(t) + Z_i(t)) \\ &\leq V\beta^{\max} - (Q_i(t) + Z_i(t)).\end{aligned} \tag{45}$$

Similarly, we can have

$$\frac{\partial P4(t)}{\partial r_i^q(t)} = -(Q_i(t) + Z_i(t)), \tag{46}$$

$$\frac{\partial P4(t)}{\partial d_i^q(t)} = -(X_i(t) + Q_i(t) + Z_i(t)). \tag{47}$$

Since $X_i(t) \geq -V\beta^{\max} - D_i^{\max}$ and $Q_i(t) > (2V\beta^{\max} + D_i^{\max})/(3)$, we get

$$\begin{aligned}&\frac{\partial P4(t)}{\partial g_i^q(t)} + \frac{\partial P4(t)}{\partial r_i^q(t)} + \frac{\partial P4(t)}{\partial d_i^q(t)} \\ &\qquad \leq V\beta^{\max} - X_i(t) - 3Q_i(t) - 3Z_i(t) < 0.\end{aligned}$$

Thus, our dynamic energy scheme that minimizes $P4(t)$ will choose $y_i(t)$ to be its maximum value. Since $y_i(t) = d_i^q(t) + g_i^q(t) + r_i^q(t)$ where $d_i^q(t) \leq D_i^{\max}$, $g_i^q(t) \leq G_i^{\max}$, and $r_i^q(t) \leq e_i(t)$, the maximum of $y_i(t)$, denoted by $y_i^{\max}(t)$, is $y_i^{\max}(t) = D_i^{\max} + G_i^{\max} + e_i(t)$.

- If $Q_i(t) \geq y_i^{\max}(t)$, we have $Q_i(t+1) = Q_i(t) - y_i^{\max}(t) + T_i(t)$. Since $T_i(t) \leq T_i^{\max} \leq G_i^{\max}$, we get $T_i(t) \leq y_i^{\max}(t)$ and hence $Q_i(t+1) \leq Q_i(t) \leq (2V\beta^{\max} + D_i^{\max})/(3) + T_i^{\max}$.
- If $Q_i(t) < y_i^{\max}(t)$, we have $Q_i(t+1) = T_i(t) \leq T_i^{\max} \leq (2V\beta^{\max} + D_i^{\max})/(3) + T_i^{\max}$.

As a result, (42) holds for all $t \geq 0$.

Next, we prove (43) by induction. Note that (43) holds for $t = 0$. Assume that (43) holds in time slot t. In what follows, we show that (43) also holds in time slot $t + 1$.

First, $0 \leq Z_i(t) \leq (2V\beta^{\max} + D_i^{\max})/(3)$. According to (34), we have $Z_i(t+1) \leq \max[Z_i(t) - y_i(t) + \epsilon_i, \epsilon_i]$. Thus, we get $Z_i(t+1) \leq Z_i(t) + \epsilon_i \leq (2V\beta^{\max} + D_i^{\max})/(3) + \epsilon_i$.

Second, $(2V\beta^{\max} + D_i^{\max})/(3) < Z_i(t) \leq (2V\beta^{\max} + D_i^{\max})/(3) + \epsilon_i$. From (45)–(47), we can have

$$\begin{aligned}&\frac{\partial P4(t)}{\partial g_i^q(t)} + \frac{\partial P4(t)}{\partial r_i^q(t)} + \frac{\partial P4(t)}{\partial d_i^q(t)} \\ &\leq V\beta^{\max} - X_i(t) - 3Q_i(t) - 3Z_i(t) \\ &\leq 2V\beta^{\max} + D_i^{\max} - 3Z_i(t) \\ &< 0\end{aligned} \tag{48}$$

due to $X_i(t) \geq -V\beta^{\max} - D_i^{\max}$ and $Q_i(t) \geq 0$. Thus, our dynamic energy scheme minimizing $P4(t)$ will choose $y_i(t) = y_i^{\max}(t) = D_i^{\max} + G_i^{\max} + e_i(t)$ as shown above.

- If $Z_i(t) \geq y_i^{\max}(t)$, we have $Z_i(t+1) \leq Z_i(t) - y_i^{\max}(t) + \epsilon_i$. Since $\epsilon_i \leq G_i^{\max}$, we get $T_i(t+1) \leq Z_i(t) \leq (2V\beta^{\max} + D_i^{\max})/(3) + \epsilon_i$.
- If $Q_i(t) < y_i^{\max}(t)$, we have $Z_i(t+1) = \epsilon_i \leq (2V\beta^{\max} + D_i^{\max})/(3) + \epsilon_i$.

Therefore, (43) holds for all $t \geq 0$.

b) We prove b) by induction. Assume that for an arbitrary user i, (10) holds in time slot t. Then, we consider the following cases to prove that (10) also holds in time slot $t + 1$.

First, $0 \leq B_i(t) < D_i^{\max}$. This case is identical to the first case of Theorem 1a. Thus, our energy management scheme takes control decisions c_i^r and c_i^g, such that $C_i(t) = c_i^r(t) + c_i^g(t) = C_i^{\max}$. Due to constraint (9), we also have $D_i(t) = 0$. Thus, according to (2), we get $B_i(t+1) = B_i(t) + C_i^{\max}$ and

$$0 \leq B_i(t+1) \leq D_i^{\max} + C_i^{\max} \leq B_i^{\max}$$

due to constraint (13).

Second, $D_i^{\max} \leq B_i(t) \leq V\beta^{\max} + D_i^{\max} + Q_i^{\max} + Z_i^{\max}$. Since

$$V \leq V^{\max} \leq \frac{B_i^{\max} - C_i^{\max} - D_i^{\max} - N_i}{\beta^{\max}},$$

according to (42) and (43), we have

$$\begin{aligned}&B_i(t) \\ &\leq B_i^{\max} - C_i^{\max} + \frac{4V\beta^{\max} + 2D_i^{\max} + 3T_i^{\max} + 3\epsilon_i}{3} - N_i \\ &\leq B_i^{\max} - C_i^{\max} \\ &\quad + \frac{4B_i^{\max} - 4C_i^{\max} - 2D_i^{\max} + 3T_i^{\max} + 3\epsilon_i - 7N_i}{3} \\ &= B_i^{\max} - C_i^{\max}.\end{aligned}$$

Thus, according to (2), we can obtain

$$B_i(t+1) \leq B_i^{\max} - C_i^{\max} + C_i(t) - D_i(t) \leq B_i^{\max}$$
and
$$B_i(t+1) \geq D_i^{\max} + C_i(t) - D_i(t) \geq 0.$$

Third, $V\beta^{\max} + D_i^{\max} + Q_i^{\max} + Z_i^{\max} < B_i(t) \leq B_i^{\max}$. Note that we have shown above that $V\beta^{\max} + D_i^{\max} + Q_i^{\max} + Z_i^{\max} \leq B_i^{\max} - C_i^{\max} < B_i^{\max}$. The partial derivative of the objective function of **P4** with respect to $d_i^g(t)$ is

$$\begin{aligned}\frac{\partial P4(t)}{\partial d_i^g(t)} &= -V\frac{\partial U(t)}{\partial d_i^g(t)} - X_i(t) \\ &\leq -V\beta^{\min} - B_i(t) + V\beta^{\max} + D_i^{\max} \\ &\leq -V\beta^{\min} - Q_i^{\max} - Z_i^{\max} \\ &< 0.\end{aligned}$$

Similarly, we can also get that $\partial P4(t)/\partial d_i^l(t) < 0$. The partial derivative of the objective function of **P4** with respect to $d_i^q(t)$ is

$$\begin{aligned}\frac{\partial P4(t)}{\partial d_i^q(t)} &= -(X_i(t) + Q_i(t) + Z_i(t)) \\ &= -B_i(t) + V\beta^{\max} + D_i^{\max} - Q_i(t) - Z_i(t) \\ &\leq -V\beta^{\max} - D_i^{\max} - Q_i^{\max} - Z_i^{\max} \\ &\quad + V\beta^{\max} + D_i^{\max} - Q_i(t) - Z_i(t) \\ &< 0.\end{aligned}$$

After minimizing $P4(t)$, our energy management scheme results in control decisions that satisfy $D_i(t) = d_i^g(t) + d_i^l(t) +$

$d_i^q(t) = D_i^{\max}$. Due to constraint (9), we have $C_i(t) = 0$. Thus, according to (2), we get $B_i(t+1) = B_i(t) - D_i^{\max}$ and hence

$$0 \le B_i(t+1) \le B_i^{\max} - D_i^{\max} \le B_i^{\max}.$$

Therefore, we can see that (10) holds for all $t \ge 0$.

c) The result (44) directly follows Lemma 2.

d) Part a) has shown that constraint (10) holds. The same as Theorem 1b, we can show that (25) and (26) hold. Part b) of this theorem has shown that DT load demands can be served before user-defined deadlines. Thus, the control decisions obtained by our dynamic energy management scheme satisfy all the constraints of $\mathbf{P3}$, and hence are feasible solutions to $\mathbf{P3}$.

e) Denote by $\widehat{C_i}(t)$, $\widehat{D_i}(t)$, $\widehat{y_i}(t)$, and $\hat{U}(t)$ the results obtained by our dynamic energy management scheme in time slot t, i.e., based on the optimal solution to $\mathbf{P4}$. We also denote by $C_i^*(t)$, $D_i^*(t)$, $y_i^*(t)$, and $U^*(t)$ the results that we get for time slot t based on the optimal solution to $\mathbf{P3}$. Thus, from Lemma 3, we can have

$$\begin{aligned}
&\Delta(\mathbf{X(t)}) + V\mathbb{E}\{\widehat{U}(t)\,|\,\mathbf{\Theta}(t)\} \\
&\quad \le K + V\mathbb{E}\{\widehat{U}(t)\,|\,\mathbf{\Theta}(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{\widehat{C_i}(t) - \widehat{D_i}(t)\,|\,\mathbf{\Theta}(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} Q_i(t)\mathbb{E}\{T_i(t) - \widehat{y_i}(t)\,|\,\mathbf{\Theta}(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} Z_i(t)\mathbb{E}\{\epsilon_i - \widehat{y_i}(t)\,|\,\mathbf{\Theta}(t)\} \\
&\quad \le K + V\mathbb{E}\{U^*(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} X_i(t)\mathbb{E}\{C_i^*(t) - D_i^*(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} Q_i(t)\mathbb{E}\{T_i(t) - y_i^*(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} Z_i(t)\mathbb{E}\{\epsilon_i - y_i^*(t)\}
\end{aligned}$$

Note that the second step is based on the fact that the optimal solutions to $\mathbf{P3}$ are obtained independent of the queue state $\mathbf{\Theta}(t)$.

Besides, since the system state $\mathbf{S}(t)$ is i.i.d., $C_i^*(t)$, $D_i^*(t)$, and $y_i^*(t)$ are also i.i.d. stochastic processes. Similar to the proof of Theorem 1c, applying the strong law of large numbers and taking expectation of both sides, we get

$$\begin{aligned}
&\mathbb{E}\{L(\mathbf{\Theta}(t+1))\} - \mathbb{E}\{L(\mathbf{\Theta}(t))\} + V\mathbb{E}\{\widehat{U}(t)\} \\
&\quad \le K + V\mathbb{E}\{U^*(t)\} \\
&\qquad + \sum_{i\in\mathcal{I}} \left(\mathbb{E}\{X_i(t)\} \cdot \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C_i^*(t) - D_i^*(t)\} \right) \quad (49) \\
&\qquad + \sum_{i\in\mathcal{I}} \left(\mathbb{E}\{Q_i(t)\} \cdot \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{T_i(t) - y_i^*(t)\} \right) \quad (50) \\
&\qquad + \sum_{i\in\mathcal{I}} \left(\mathbb{E}\{Z_i(t)\} \cdot \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{\epsilon_i - y_i^*(t)\} \right) \quad (51)
\end{aligned}$$

We have shown by (27) that $\lim_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{C_i^*(t)\} = \lim_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{D_i^*(t)\}$. Thus, the component (49) is equal to 0. Since $Q_i(t) \le Q_i^{\max} < \infty$ for all $t \ge 0$ where $Q_i^{\max}$ is a constant defined in (28), we have $\limsup_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{Q_i(t)\} \le Q_i^{\max}$, i.e., queue $Q_i(t)$ is strongly stable [18]. Since $y_i(t) - T_i(t) \le y_i(t) \le y_i^{\max}$, we know that queue $Q_i(t)$ is also rate stable (Theorem 2.8, i.e., Strong Stability Theorem, in [18]), i.e., $\limsup_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{T_i(t)\} \le \limsup_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{y_i^*(t)\}$, which means the component (50) is no larger than 0. Similarly, since $y_i(t) - \epsilon_i\mathbf{1}_{Q_i(t)>0} \le y_i^{\max}$, queue $Z_i(t)$ is rate stable and $\limsup_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{\epsilon_i\mathbf{1}_{Q_i(t)>0}\} \le \limsup_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{y_i^*(t)\}$. Thus, we have $\lim_{T\to\infty}(1/T)\sum_{t=0}^{T-1}\mathbb{E}\{\epsilon_i - y_i^*(t)\} \le 0$, i.e., the component (51) is no larger than 0. Therefore, we have

$$\begin{aligned}
&\mathbb{E}\{L(\mathbf{\Theta}(t+1))\} - \mathbb{E}\{L(\mathbf{\Theta}(t))\} + V\mathbb{E}\{\widehat{U}(t)\} \\
&\qquad\qquad \le K + V\mathbb{E}\{U^*(t)\}
\end{aligned}$$

Similar to the proof of Theorem 1c, summing the above inequality over all the time slots $t \in \{0, 1, 2, \ldots, T-1\}$, dividing both sides by VT, and taking limits as $T \to \infty$, we can get $\mathbf{P3}^* \ge \mathbf{P4}^* - K/V$.

Besides, as shown in d), the optimal solution to $\mathbf{P4}$ is also a feasible solution to $\mathbf{P3}$. Thus, the value of the objective function of $\mathbf{P3}$ calculated based on the optimal solution to $\mathbf{P4}$, i.e., $\mathbf{P4}^*$, is an upper bound on $\mathbf{P3}^*$, i.e., $\mathbf{P3}^* \le \mathbf{P4}^*$.

We have now completed the proof.

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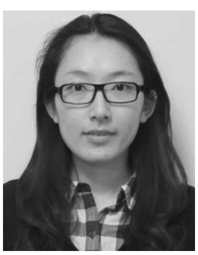

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