Economic-Robust Transmission Opportunity Auction in Multi-hop Wireless Networks

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***Abstract*—The rapid growth of wireless devices and services exacerbates the problem of spectrum scarcity in wireless networks. Recently, spectrum auction has emerged as one of the most promising techniques to enhance spectrum utilization and mitigate this problem. Although there exist some works studying spectrum auction, most of them are designed for single-hop communications, and it is usually not clear whom a winning user communicates with. Moreover, most previous auction schemes only focus on satisfying the incentive compatibility property, also called truthfulness, but ignore another two critical properties: individual rationality, and budget balance. Thus, they may not be economic-robust. In this paper, we propose a transmission opportunity auction scheme, called TOA, which can support multi-hop data traffic, ensure economic-robustness, and generate high revenue for the auctioneer. Specifically, in TOA, instead of spectrum bands as in traditional spectrum auction schemes, users bid for transmission opportunities (TOs). A TO is defined as the permit of data transmission on a specific link using a certain band, i.e., a link-band pair. The TOA scheme is composed of three procedures: TO allocation, TO scheduling, and pricing, which are performed sequentially and iteratively until the aforementioned goals are reached. We prove that TOA is economic-robust, and conduct extensive simulations to show its effectiveness and efficiency.**

I. Introduction

The past few years have witnessed substantial growth of wireless devices and services, which, on the other hand, makes spectrum an even more scare resource in wireless networks. Traditional spectrum allocation was conducted in a static manner, resulting in inefficient spectrum utilization. Recently, spectrum sharing through a dynamic real-time secondary spectrum auction market has been proposed to enhance spectrum utilization and mitigate the problem of spectrum scarcity. In such a market, a spectrum owner or primary user (PU) leases its idle licensed spectrum bands to secondary users (SUs) to gain profits. SUs, who do not have their own spectrums but need to deliver data traffic, compete for spectrum bands and pay for them if they succeed in the spectrum auction.

In the literature, there have been some works studying spectrum auction in wireless networks. Unfortunately, most of them [1]–[11] are only suitable for single-hop data transmission. In particular, in these schemes, each user bids and is allowed to use the purchased spectrum for communications if it wins. However, there are two problems: first, this is only for single-hop communications, and second, it is not clear whom a winning user communicates with (the receiver is not clearly specified). Thus, the network performance can be poor. Zhu *et al.* [12] discuss spectrum auction for multi-hop data delivery. But they assume that each secondary network only has one flow, and do not consider time domain scheduling when utilizing the spectrums.

Moreover, in addition to fulfilling SUs' traffic demands, auction schemes need to satisfy certain economic properties. Specifically, incentive compatibility (IC) (also called truthfulness or strategy-proof), individual rationality (IR), and budget balance (BB) are three of the most critical properties in auction design. An auction is called economic-robust [8], [13] if all these three properties are preserved. It has been shown both theoretically and practically that an auction could be vulnerable to market manipulation and produce very poor outcomes if those properties are not guaranteed [14]. Most previous auction schemes focus on IC only, but do not necessarily satisfy the other two properties.

In this paper, we aim to design an economic-robust auction scheme for multi-hop wireless networks. In particular, we consider an auction market where a PU acts as an auctioneer and leases its idle licensed spectrum bands to some SUs, which are deployed by a secondary service provider (SSP) to fulfill certain purposes, such as data delivery, data collection, and object tracking. SUs may need to transmit data to their destinations that are multiple hops away. To deliver the data traffic, the SSP asks all the SUs to submit bids to the auctioneer. If some SUs win, they pay a price to the auctioneer and relay data traffic for each other using the spectrum purchased. The SSP finally pays back all the winning SUs, and lets them gain some profits so that they are motivated to participate in the auction.

To support multi-hop data traffic, ensure economic-robustness, and generate high revenue for the auctioneer, we propose a transmission opportunity auction scheme, called TOA. In TOA, instead of spectrum bands as in traditional spectrum auction schemes, SUs bid for transmission opportunities (TOs). A TO is defined as the permit of data transmission on a specific link using a certain band, i.e., a link-band pair. The TOA scheme is mainly composed of three procedures: TO allocation, TO scheduling, and pricing. These three procedures are performed sequentially and iteratively until the aforementioned goals are reached. Specifically, in TO allocation, in

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each iteration the auctioneer solves a TO allocation (TO-AL) optimization problem to find out the link-band pairs (i.e., TOs) that can be active at the same time and have the highest total bid. It considers the set of the transmitters in these TOs as a winning virtual bidder group (VBG). In TO scheduling, the auctioneer formulates a minimum length scheduling problem, called TO scheduling (TO-SC), to see if the winning VBGs found so far can support the traffic demand in the network by exploring scheduling (in both time and frequency domains) and routing. If the minimum scheduling length is larger than 1, it means that the current winning VBGs cannot support the traffic demand, and the auctioneer needs to find another VBG through TO-AL again. Otherwise, the auctioneer can then determine the clearing price for each winning VBG and SU, and computes its own revenue. The auctioneer finally chooses the iteration, i.e., the winning VBGs, that can generate the highest revenue among the results it obtains.

Moreover, notice that our auction scheme TOA is developed based on second-price sealed-bid auction [15]. We prove that TOA is economic-robust for VBGs and individual SUs.

The rest of this paper is organized as follows. We discuss related work in Section II. The problem formulation is presented in Section III. We detail the proposed transmission opportunity auction (TOA) scheme in Section IV, and prove the economic-robustness of TOA in Section V, respectively. We conduct simulations in Section VI to evaluate the performance of TOA. We finally conclude this paper in Section VII.

II. Related Work

Auction has been employed by the Federal Communications Commission (FCC) to efficiently allocate spectrum resources [16]. Based on this idea, some works propose to apply auction to spectrum sharing in wireless networks.

Kloeck *et al.* [1] consider a multi-unit sealed-bid auction for efficient spectrum allocation. Huang *et al.* [11] propose an auction mechanism which allows users to bid for their transmission power to efficiently share the spectrum. Gandhi *et al.* [2] design an auction scheme considering spectrum reuse in wireless networks. However, all the above works ignore the possible strategic behavior of bidders. Zhou *et al.* [10] then propose a truthful spectrum auction scheme VERITAS with greedy channel assignment and critical value based pricing. Jia et al. [3] discuss how to generate maximum expected revenue, which is an alternate goal of maximum social welfare, in spectrum auction while satisfying the truthfulness property. In order to further improve the expected revenue, Al-Ayyoub *et al.* [6] design a color-based channel allocation scheme. Taking fairness in channel allocation into account, Gopinathan *et al.* [7] develop a truthful auction protocol by applying linear programming techniques to balance the social welfare and max-min fairness in secondary spectrum markets. In addition to single-sided auction, some works employ double auction in spectrum market, where multiple spectrum owners compete with each other to sell idle spectrums for profit. Zhou and Zheng [8] propose a framework TRUST for truthful double spectrum auction enabling spectrum reuse. Wang *et al.* [9] design a truthful double auction scheme considering that spectrums are tradable only within their licensed areas. However, these two works assume each seller can only sell one channel and each buyer can buy one channel at most. This limits the utility of both buyers and sellers, as well as the revenue of the auctioneer.

Most importantly, all the above works are only suitable for single-hop data transmission. Although Zhu *et al.* [12] discuss spectrum auction for multi-hop data delivery, they assume that each secondary network only has one flow, and do not consider time domain scheduling when utilizing the spectrums.

III. Problem Formulation

A. System Model

We consider an auction market where a spectrum owner or primary user (PU) acts as an auctioneer and leases its idle licensed spectrum bands $\mathcal{M} = \{1, 2, ..., m, ..., M\}$ to secondary users (SUs) $\mathcal{N} = \{1, 2, ...n, ..., N\}$. The SUs are deployed by a secondary service provider (SSP) to fulfill some purposes such as data delivery, data collection, and object tracking. In this study, we assume that each SU is equipped with one radio, which means it cannot transmit and receive simultaneously. Suppose there are a set of $\mathcal{L} = \{1, 2, ..., l, ...L\}$ sessions in the secondary network. We let $s(l)$, $d(l)$, and $r(l)$ denote the source node, destination node, and traffic demand of session $l \in \mathcal{L}$, respectively. To deliver the traffics, the SSP asks all the SUs to submit bids to the auctioneer for transmission opportunities (TOs), each of which is defined as the permit of data transmission on a specific link using a certain band, i.e., a link-band pair. If some SUs win, they pay a price to the auctioneer and relay data traffic for each other with obtained TOs. The SSP finally pays back all the winning SUs and lets them gain some profits.

Given the network topology, the PU can construct a conflict graph denoted by $G(V, E)$, where V is the vertex set and E is the edge set. In particular, each vertex corresponds to a link-band pair denoted by $((i, j), m)$, where $i \in \mathcal{N}$, $j \in \mathcal{T}_i^m$, and $m \in \mathcal{M}$. Here, $\mathcal{T}_i^m$ is the set of SUs within SU i's transmission range on band m. Besides, two vertices in V are connected with an undirected edge if the corresponding link-band pairs interfere with each other, i.e., if any of the following conditions is true:

- The receiving SU in one link-band pair is within the interference range of the transmitting SU in another link-band pair, given that the both of them are using the same band;
- The two link-band pairs have at least one node in common.

In this conflict graph, an independent set (IS) is a set in which each element is a link-band pair standing for a transmission, and all the elements (or transmissions) can be carried out successfully at the same time. If adding any more link-band pairs into an IS results in a non-independent one, this IS is defined as a maximum independent set (MIS). We denote the set of all the MISs by $\mathcal{I} = \{\mathcal{I}_1, \mathcal{I}_2, ...\mathcal{I}_q, ..., \mathcal{I}_Q\}$, where $Q = |\mathcal{I}|$, and $\mathcal{I}_q \subseteq V$ for $1 \leq q \leq Q$. We will show later that we do not really need to find all the MISs. We denote the

MIS $\mathcal{I}_q$'s time share (out of unit time 1) to be active by λ_q ($\lambda_q \geq 0$). Therefore, if all the data traffics in the network can be supported, we have $\sum_{q=1}^{Q} \lambda_q \leq 1$. Besides, we let $c_{ij}^m(\mathcal{I}_q)$ be the instantaneous transmission rate of the link-band pair $((i,j),m)$ when MIS $\mathcal{I}_q$ is active. Thus, $c_{ij}^m(\mathcal{I}_q)$ is equal to 0 when $((i,j),m) \notin \mathcal{I}_q$, and the capacity of $((i,j),m)$, i.e., c_{ij}^m, otherwise, which will be introduced soon.

Moreover, we denote an SU i's real valuation of and bid price for unit instantaneous transmission rate by v_i and p_i, respectively. Note that v_i can be the rewards SU i receives from the SSP if it wins. In an auction, SUs submit their bids p_i's in a sealed manner, so that no one has access to any information about the others' bids. After the auctioneer receives all the bids, it divides the bidders into different virtual bidder groups (VBGs), each of which is the set of transmitters of all link-band pairs in one MIS. Similarly, we denote the set of all the VBGs by $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_q, ..., \mathcal{G}_Q\}$. Obviously, we have $|\mathcal{I}| = |\mathcal{G}| = Q$. Then, with SUs' unit valuations and unit bid prices, the auctioneer can then calculate SUs' *equivalent valuations* of and *equivalent bids* for different TOs. Notice that in one MIS, any SU can have at most one TO. Let v_i^q denote SU i's equivalent valuation of the TO it can obtain from $\mathcal{I}_q$. Then, we can have $v_i^q = v_i \cdot \sum_{m\in\mathcal{M}} \sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_q)$. Accordingly, SU i's equivalent bid for the same TO, denoted by b_i^q, can be calculated as $b_i^q = p_i \cdot \sum_{m\in\mathcal{M}} \sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_q)$.

The auctioneer considers VBGs as virtual bidders in the auction.The *virtual bid* from a VBG is the sum of all SUs' equivalent bids in the group. In particular, let B_q denote the virtual bid from VBG $\mathcal{G}_q$ ($1 \leq q \leq Q$). Then, we have $B_q = \sum_{i\in\mathcal{G}_q} b_i^q$. Denote by $\mathbf{B_{-q}}$ the vector of the virtual bids from the other VBGs $\mathcal{G}/\mathcal{G}_q$. Thus, the entire bid price set, denoted by $\mathbf{B}$, is $\mathbf{B} = (B_q, \mathbf{B_{-q}})$. Besides, denote by $\mathcal{G}_W$ the set of the winning VBGs, and $\mathcal{I}_W$ the set of the corresponding winning MISs. Notice that an SU can be involved in multiple winning VBGs. Thus, SU i's *total equivalent bid* for the TOs it obtains, denoted by b_i, is equal to $b_i = \sum_{\mathcal{G}_q\in\mathcal{G}_W} b_i^q$. Note that $b_i^q = 0$ if $i \notin \mathcal{G}_q$.

B. Objective of Auction Design

The design of auction schemes heavily depends on the desired properties. In this paper, we assume that all SUs are strategic in the sense that they may manipulate their bids to obtain favorable outcomes. Denote by c_i ($i \in \mathcal{N}$) the clearing price the auctioneer charges SU i for unit instantaneous transmission rate. We aim to design an auction scheme that can satisfy three of the most important economic requirements: Incentive Compatibility (IC), Individual Rationality (IR) and Budget Balance (BB), which are defined as follows:

- **Incentive Compatibility (IC)**: The utility function of SU i ($i \in \mathcal{N}$), is a function of all the bids:
$$u_i(p_i, \mathbf{p_{-i}}) = \begin{cases} \left[\sum_{\mathcal{G}_q\in\mathcal{G}_W} \sum_{m\in\mathcal{M}} \sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_q)\right] \\ \cdot(v_i - c_i), \text{ if } i \text{ wins with unit bid } p_i, \\ 0, \text{ otherwise.} \end{cases} \tag{1}$$
where $\mathbf{p_{-i}}$ denotes the vector of bids from the other SUs. Thus, an auction is IC if for any SU i ($i \in \mathcal{N}$) with any $p_i \neq v_i$ while the others' bids are fixed, we have
$$u_i(p_i, \mathbf{p_{-i}}) \leq u_i(v_i, \mathbf{p_{-i}}). \tag{2}$$
- **Individual Rationality (IR)**: An auction is IR, if no bidder is charged higher than its bid in the auction, i.e., $c_i \leq p_i$ for all $i \in \mathcal{N}$.
- **Budget Balanced (BB)**: To make the auction self-sustained without any external subsidies, the generated revenue of the auctioneer, i.e., the PU, is required to be non-negative.

We say an auction is *economic-robust* [8], [13] if it is incentive compatible, individually rational and budget balanced. Since in this paper, we consider that the PU leases its own idle spectrum bands without causing quality degradation to its own services, the PU's revenue is the total payment received from the winning SUs, which is always non-negative. Thus, our auction scheme is always BB. We will focus on achieving IC and IR in our auction scheme design. Moreover, an auction scheme is said to be *system-efficient* if the revenue of auctioneer is maximized. Unfortunately, according to the impossibility theorem demonstrated in [17], an auction cannot be economic-robust and system-efficient at the same time. Therefore, in this study we aim to design an economic-robust auction, while try to generate high revenue for the auctioneer.

C. Transmission Opportunity's Capacity

Suppose the power spectral density of SU i on band m is a constant and denoted by P_i^m. A widely used model [18] for power propagation gain between SU i and SU j, denoted by g_{ij}, is $g_{i,j} = C \cdot [d(i,j)]^{-\gamma}$, where i and j also denote the positions of SU i and SU j, respectively, $d(i,j)$ refers to the Euclidean distance between i and j, γ is the path loss factor, and C is a constant related to the antenna profiles of the transmitter and the receiver, wavelength, and so on. We assume that the data transmission is successful only if the received power spectral density at the receiver exceeds a threshold P_T^m. Meanwhile, we assume interference becomes non-negligible only if it produces a power spectral density over a threshold of P_I^m at the receiver. Thus, the transmission range of SU i on band m is $R_T^{i,m} = (CP_i^m/P_T^m)^{1/\gamma}$, which comes from $C(R_T^{i,m})^{-\gamma} \cdot P_i^m = P_T^m$. Similarly, based on the interference threshold $P_I^m (P_I^m \leq P_T^m)$, the interference range of SU i is $R_I^{i,m} = (CP_i^m/P_I^m)^{1/\gamma}$, which is no smaller than $R_T^{i,m}$. Thus, different SUs may have different transmission ranges/interference ranges on different channels with different transmission power.

In addition, according to the Shannon-Hartley theorem, if SU i sends data to SU j on link (i,j) using band m, the capacity of the TO, i.e., link-band pair $((i,j),m)$, is
$$c_{ij}^m = W^m \log_2\left(1 + \frac{g_{ij}P_i^m}{\eta}\right), \tag{3}$$
where η is the thermal noise at the receiver. Note that the denominator inside the log function only contains η. This is because of one of our interference constraints, i.e., when node i is transmitting to node j on band m, all the other neighbors of node j within its interference range are prohibited from

using this band. We will address the interference constraints in details in the following section.

IV. Transmission Opportunity Auction

In this section, we introduce our proposed transmission opportunity auction scheme, called TOA. Recall that in the network there are SUs who need to deliver data traffic to their destinations that are multiple hops away. Thus, the objective of TOA is to choose MISs, and hence VBGs, which can support such traffics and bring high revenue to the auctioneer. Meanwhile, TOA should be economic-robust. In general, the TOA scheme is composed of three procedures: TO allocation, TO scheduling, and pricing. These three procedures are performed sequentially and iteratively until our goals are reached. In what follows, we detail the design of the three procedures, respectively.

A. Transmission Opportunity Allocation

At the beginning of TO auction, each SU i ($i \in \mathcal{N}$) submits its unit bid price p_i to the auctioneer. Then, as mentioned before, the auctioneer can calculate SU i's equivalent bids b_i^q for the TO it obtains from a VBG $\mathcal{G}_q$, and the virtual bid from $\mathcal{G}_q$, which is

$$B_q = \sum_{i \in \mathcal{G}_q} b_i^q = \sum_{i \in \mathcal{G}_q} \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_q) \cdot p_i. \qquad (4)$$

Note that as explained above, an auction cannot be economically robust and system-efficient at the same time, and in this study we aim to design an economic-robust auction. Thus, the objective of TO allocation is to find out one winning MIS, which corresponds to a winning VBG, that maximizes the virtual bid B_q in each iteration in a monotonic manner. In particular, we will find the VBG with the highest virtual bid in the first iteration, the one with the second highest virtual bid in the second iteration, and so on and so forth until the iteration ends. Such VBGs (MISs) are considered as winning VBGs (MISs) denoted by $\mathcal{G}_W$ ($\mathcal{I}_W$). We will show in SectionV-A that a monotonic TO allocation procedure is critical in achieving the IC and IR properties.

Before formulating the optimization problem, we first list several constraints as follows.

Notice that in the procedure of TO allocation, we do not assume that we know all the MISs, finding which is in fact an NP-complete problem. We denote

$$s_{ij}^m = \begin{cases} 1, \text{ if } i \text{ can transmit to } j \text{ on band } m, \\ 0, \text{ otherwise.} \end{cases}$$

Since an SU is not able to transmit to or receive from multiple SUs on the same frequency band, we have

$$\sum_{j \in \mathcal{T}_i^m} s_{ij}^m \leq 1, \text{ and } \sum_{\{i | j \in \mathcal{T}_i^m\}} s_{ij}^m \leq 1. \qquad (5)$$

Besides, an SU cannot use the same frequency band for transmission and reception, due to "self-interference" at physical layer, i.e.,

$$\sum_{\{i | j \in \mathcal{T}_i^m\}} s_{ij}^m + \sum_{q \in \mathcal{T}_j^m} s_{jq}^m \leq 1. \qquad (6)$$

Moreover, recall that in this study, we consider each SU is only equipped with a single radio, which means each SU can only transmit or receive on one frequency band at a time. Thus, we can have

$$\sum_{m \in \mathcal{M}} \sum_{\{i | j \in \mathcal{T}_i^m\}} s_{ij}^m + \sum_{m \in \mathcal{M}} \sum_{q \in \mathcal{T}_j^m} s_{jq}^m \leq 1. \qquad (7)$$

Notice that (5)-(6) will hold whenever (7) holds.

In addition to the above constraints at a certain SU, there are also constraints due to potential interference among the SUs. In particular, for a frequency band m, if SU i uses this band for transmitting data to a neighboring SU $j \in \mathcal{T}_i^m$, then any other SUs that can interfere with SU j's reception should not use this band. To model this constraint, we denote by $\mathcal{P}_j^m$ the set of SUs that can interfere with SU j's reception on band m, i.e.,

$$\mathcal{P}_j^m = \{p | d(p, j) \leq R_I^{p,m}, p \neq j, \mathcal{T}_p^m \neq \emptyset\}.$$

The physical meaning of $\mathcal{T}_p^m \neq \emptyset$ in the above definition is that SU p has at least one neighbor to which it may transmit data and hence cause interference to SU j's reception. Therefore, we have

$$\sum_{\{i | j \in \mathcal{T}_i^m\}} s_{ij}^m + \sum_{q \in \mathcal{T}_p^m} s_{pq}^m \leq 1 \qquad (\forall p \in \mathcal{P}_j^m). \qquad (8)$$

Moreover, recall that we need find the t-th highest virtual bid in the t-th iteration. Thus, in the t-th ($t \geq 2$) iteration, we need find the VBG giving the highest virtual bid with the previously found $t-1$ VBGs being excluded. Letting $\mathcal{I}_{W,t}$ and $\mathcal{G}_{W,t}$ denote the MIS and the corresponding VBG that we find in the t-th iteration, respectively, we have

$$\sum_{((i,j),m) \in \mathcal{I}_{W,\tau}} s_{ij}^m < |\mathcal{I}_{W,\tau}|, \quad 1 \leq \tau \leq t-1, \qquad (9)$$

$$\sum_{((i,j),m) \notin \mathcal{I}_{W,\tau}} s_{ij}^m \geq 1, \quad 1 \leq \tau \leq t-1, \qquad (10)$$

where $|\mathcal{I}_{W,\tau}|$ is the number of elements contained in $\mathcal{I}_{W,\tau}$. (9) means that all the link-band pairs in any of the previously found $t-1$ MISs cannot be selected at the same time in the t-th iteration, which excludes the previous $t-1$ MISs. (10) means that the newly found MIS should contain at least one different link-band pair from any of the previously found $t-1$ MISs.

Consequently, according to the above constraints, the TO allocation (TO-AL) optimization problem finding the VBG with the t-th highest virtual bid in the t-th iteration can be formulated as follows:

$$\textbf{TO-AL:} \qquad \textbf{Maximize} \quad \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{T}_i} \sum_{m \in \mathcal{M}_i \cap \mathcal{M}_j} c_{ij}^m s_{ij}^m \cdot p_i$$

$$\textbf{s.t.} \quad \text{Equations } (7) - (10)$$

$$s_{ij}^m = 0 \quad \text{or} \quad 1$$

where s_{ij}^m's are the optimization variables, c_{ij}^m's are calculated according to (3), p_i are known constants received from the

SUs. Note that (9) and (10) make sure the newly found IS in t-th iteration is an MIS and it is different from any MIS found in previous $t-1$ iterations. Besides, (9) is in fact always satisfied as long as (10) holds. Since s_{ij}^m can only take value of 0 or 1, TO-AL is a Binary Integer Programming (BIP) problem, which can be solved by applying the traditional *branch-and-bound* or *branch-and-cut* [19] approach.

B. Transmission Opportunity Scheduling

In this paper, we assume strict allocation [3] in TO auction, i.e., a source node pays the auctioneer only if its traffic demand is fully satisfied. Thus, the auctioneer needs to find an optimal way to utilize those winning MISs, trying to deliver all source nodes' traffic by exploring joint scheduling and routing.

Denote the set of the winning MISs found up to the t-th iteration by $\mathcal{I}_W^t = \cup_{\tau=1}^t \mathcal{I}_{W,\tau}$. Note that $|\mathcal{I}_W^t| = t$. Letting $f_{ij}(l)$ denote the flow rate of traffic l over link (i,j), where $i \in \mathcal{N}$, $l \in \mathcal{L}$, and $j \in \mathcal{T}_i$ given $\mathcal{T}_i = \cup_{m\in\mathcal{M}} \mathcal{T}_i^m$, the scheduling of the MISs should satisfy the following:

$$\sum_{l\in\mathcal{L}} f_{ij}(l) \leq \sum_{q=1}^{t} \lambda_q \sum_{m\in\mathcal{M}} c_{ij}^m(\mathcal{I}_q). \tag{11}$$

We then give routing constraints in the following. Recall that a source SU may need a number of relay nodes to relay its data packets toward the intended destination node. Since routing packets along a single path may not be able to fully take advantage of the local available channels, in this study, we employ multi-path routing to deliver packets more effectively and efficiently.

In particular, if SU i is the source of session l, i.e., $i = s(l)$, then we have the following constraints:

$$\sum_{j\neq s(l), s(l)\in\mathcal{T}_j} f_{js(l)}(l) = 0, \tag{12}$$

$$\sum_{j\neq s(l), j\in\mathcal{T}_{s(l)}} f_{s(l)j}(l) = r(l). \tag{13}$$

The first constraint means that the incoming data rate of session l at its source node is 0. The second constraint means that the traffic for session l may be delivered through multiple nodes on multiple paths, and the total data rates on all outgoing links are equal to the corresponding traffic demand $r(l)$.

If SU i is an intermediate relay node for session l, i.e., $i \neq s(l)$ and $i \neq d(l)$, then

$$\sum_{j\neq s(l), j\in\mathcal{T}_i} f_{ij}(l) = \sum_{p\neq d(l), i\in\mathcal{T}_p} f_{pi}(l), \tag{14}$$

which indicates that the total incoming data rates at a relay node are equal to its total outgoing data rates for the same session.

Moreover, if SU i is the destination node of session l, i.e., $i = d(l)$, then we have

$$\sum_{j\neq d(l), j\in\mathcal{T}_{d(l)}} f_{d(l)j}(l) = 0, \tag{15}$$

$$\sum_{p\neq d(l), d(l)\in\mathcal{T}_p} f_{pd(l)}(l) = r(l). \tag{16}$$

The first constraint means the total outgoing data rate for session l at its destination $d(l)$ is 0, while the second constraint indicates that the total incoming data rate for session l at the destination $d(l)$ is equal to the corresponding traffic demand $r(l)$.

Thus, based on the constraints mentioned above, the TO scheduling (TO-SC) optimization problem in the t-th iteration can be formulated as follows:

$$\begin{aligned} \textbf{TO-SC:} \quad & \textbf{Minimize} \quad \sum_{q=1}^{t} \lambda_q \\ & \textbf{s.t.} \quad \text{Equations } (11) - (16) \\ & \lambda_q \geq 0 \ (1 \leq q \leq t) \\ & f_{ij}(l) \geq 0 \ (i \in \mathcal{N}, j \in \mathcal{T}_i, l \in \mathcal{L}) \end{aligned}$$

The formulated optimization problem is a linear programming (LP) problem, which can be easily solved by using the simplex method [20]. The optimal result of TO-SC indicates whether the current winning MISs are enough to support the traffic demand. Specifically, If the optimal objective function is no larger than 1, then the traffic can be supported. The solution also shows how to schedule the MISs and route the traffics. Then, the auctioneer continues to perform pricing as introduced next. Otherwise, it means that the current winning MISs cannot satisfy the traffic demand. Thus, the auctioneer does not need to perform pricing and another winning MIS is needed from TO-AL.

C. Pricing

In an iteration, if the minimum scheduling length $\sum_{q=1}^t \lambda_q$ is no larger than 1, given the winning MISs $\mathcal{I}_W$ and their schedules, the auctioneer can then determine the clearing price for each SU. The pricing procedure consists of two steps: determining the clearing price for each winning VBG, and determining the clearing price for each winning SU.

Denote the number of iterations the auctioneer takes to get $\sum_{q=1}^t \lambda_q \leq 1$ for the first time by T_0. To maintain the economic properties and take spectrum utilization into consideration, we determine the clearing price for each winning VBG in the t-th iteration, denoted by C_t, as follows:

$$C_t = \max\left\{B_t \cdot \sum_{q=1}^{t} \lambda_q, \quad B_{t+1}\right\}, \text{ for } t \geq T_0,$$

where B_t is the virtual bid from the VBG found in the t-th iteration, i.e., the lowest bid among all the winning VBGs' bids, and B_{t+1} is the virtual bid from the VBG found in the $(t+1)$-th iteration, i.e., the highest bid among all the losing VBGs' bids. Notice that $\sum_{q=1}^t \lambda_q$ indicates the spectrum utilization. When it is less than 1, it means that the auctioneer can launch another auction to rent the unutilized spectrum, and hence it is reasonable to consider it in the clearing price.

With each winning VBG's clearing price defined as above, the price a winning SU i needs to pay, denoted by $C_{t,i}$, is given as follows:

$$C_{t,i} = \sum_{q=1}^{t} \left(\frac{b_i^q}{B_q} \cdot C_t\right), \text{ for } t \geq T_0.$$

Note that $\frac{b_i^q}{B_q} \cdot C_t$ is the price SU i needs to pay in VBG $\mathcal{G}_{W,q}$, and hence $C_{t,i}$ is the total clearing price for SU i.

Thus, a winning SU i's clearing price, denoted by $c_{t,i}$, is

$$c_{t,i} = \frac{C_{t,i}}{\sum_{q=1}^{t} \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q})}, \text{ for } t \geq T_0.$$

D. Iteration Termination Condition

As mentioned before, the auctioneer performs the above three procedures sequentially and iteratively. Here, we discuss when the auctioneer stops and finishes the auction process.

Notice that the auctioneer's revenue obtained in the t-th iteration, denoted by $R(t)$, is $R(t) = C_t \cdot t$. According to Moon and Moser's result [21], any graph with n vertices has at most $3^{\frac{n}{3}}$ MISs. Thus, the number of iterations does not need to exceed $3^{\frac{|V|}{3}}$, which we denote by T_a i.e., $t \leq T_a$. Recall that we denote the number of iterations the auctioneer takes to get $\sum_{q=1}^{t} \lambda_q \leq 1$ for the first time by T_0. We also define a control parameter T_b to be the maximum number of iterations the auctioneer runs beyond T_0 to calculate for its maximum revenue under the proposed auction scheme. Therefore, we have $t \leq T_0 + T_b$, and hence $t \leq \min\{T_0 + T_b, T_a\}$. We will show in our simulations that we usually only need a small number of iterations in practice. Moreover, in the case that the auctioneer finds that the SUs' traffic demands cannot be supported, the auctioneer will drop one of them each time until the remaining traffic demands can be satisfied.

After the iteration ends, the auctioneer finds the optimal iteration t^* that gives the maximum revenue $R(t)$ among all the $t - T_0 + 1$ (from T_0 to t) outcomes. Note that $R(t)$ is not equal to the maximum revenue the auctioneer can possibly get under *system-efficient* auction scheme as we explained before. Then, SU i's clearing price will be $c_{t^*,i}$.

V. Proof of Economic Properties

In this section, we first prove that our proposed auction scheme TOA is IC and IR for VBGs, and then show that those two economic properties also hold for individual SUs.

A. Proof of Economic-robustness for VBGs

According to Myerson's characterization of IC and IR sustained auction [22], if the item in the auction is monotonically allocated and the winners are charged with critical value, then the auction satisfies the IC and IR properties.

Definition 1: **Monotonic Allocation:** When others' bids, i.e., $\mathbf{B_{-q}}$ are fixed, if one bidder wins by bidding B_q, then it also wins by bidding $B'_q > B_q$.

Definition 2: **Critical Value:** Critical value is such a value that if bidders bid higher than it, then they win, and if bidders bid lower than it, then they lose.

Lemma 1: The auction items, i.e., TOs, are monotonically allocated in our auction scheme.

Proof: Since the TO allocation procedure determines a winning VBG each time by finding the one with the highest bid, the lemma directly follows. ■

Lemma 2: The clearing price C_t for each winning VBG is a critical value.

Proof: Recall that B_t is the virtual bid from the VBG found in the t-th iteration, i.e., the t-th winning VBG, and the clearing price is $C_t = \max\{B_t \cdot \sum_{q=1}^{t} \lambda_q, B_{t+1}\}$. First, if $B_t \cdot \sum_{q=1}^{t} \lambda_q \leq B_{t+1}$, then C_t is equal to B_{t+1}, which is obviously a critical value since bidders with higher bids than B_{t+1} win and those with lower bids lose. Second, if $B_t \cdot \sum_{q=1}^{t} \lambda_q > B_{t+1}$, then C_t is equal to $B_t \cdot \sum_{q=1}^{t} \lambda_q$, which is also a critical value. Thus, the clearing price is always a critical value.

■

Thus, from Lemma 1 and Lemma 2, we have the following theorem.

Theorem 1: The proposed auction scheme TOA is IC and IR, and hence economically robust for VBGs.

B. Proof of Economic-robustness for Individual SUs

Although in the previous section, we have shown that our proposed auction scheme TOA preserves IC and IR properties and hence is economically robust for VBGs, we need further prove that it also has these properties for individual SUs.

The following lemma demonstrates the monotonic allocation for SUs.

Lemma 3: When the other SUs' bids, i.e., $\mathbf{p_{-i}}$ are fixed, if SU i wins by bidding p_i, then it also wins by bidding $p'_i > p_i$.

Proof: Consider an arbitrary iteration t. If SU i is a winner up to this iteration with bid p_i, it means that i is in at least one of the t winning VBGs. Denote the winning VBG that contains SU i and wins in the q-th iteration by $\mathcal{G}_q$ ($1 \leq q \leq t$). Then, its virtual bid is

$$B_q = \sum_{j \in \mathcal{G}_q} b_j^q = \sum_{j \in (\mathcal{G}_q \setminus i)} b_j^q + b_i^q$$

where $b_i^q = p_i \cdot \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q})$. When SU i bids $p'_i > p_i$, the VBG $\mathcal{G}_q$'s new virtual bid, denoted by B'_q, is

$$B'_q = \sum_{j \in (\mathcal{G}_q \setminus i)} b_j^q + p'_i \cdot \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q}) > B_q.$$

Denote the set of the winning VBGs found up to the q-th iteration by $\mathcal{G}_W^q$, i.e., $\mathcal{G}_W^q = \cup_{\tau=1}^{q} \mathcal{G}_{W,\tau}$. For any VBG $\mathcal{G}_s$ that does not contain SU i and loses in all q iterations when i bids with p_i, i.e., $i \notin \mathcal{G}_s$ and $\mathcal{G}_s \in \mathcal{G} \setminus \mathcal{G}_W^q$, we denote its virtual bid when SU i bids with p_i and with p'_i by B_s and B'_s, respectively. Since the other SUs' bids remain the same, we have

$$B'_s = B_s \leq B_q < B'_q.$$

Therefore, the VBGs which do not contain SU i and lose in all q iterations when i bids with p_i will still lose when i bids with p'_i. Consequently, when i bids with p'_i, since the virtual bids of the VBGs containing SU i become larger, the number of VBGs containing SU i in the q wining VBGs in all q iterations gets no smaller than that when i bids with p_i. Thus, SU i still wins. ■

Using the above lemma, we are able to prove the IC property for individual SUs as follows.

Theorem 2: The proposed auction scheme TOA is IC for SUs.

Proof: Recall that to prove the IC property, we need to show that for any SU i with any $p_i \neq v_i$ while the others' bids are fixed, the condition in (2) holds.

Let $u_i(p_i, \mathbf{p_{-i}})$ and $u_i(v_i, \mathbf{p_{-i}})$ denote SU i's utility when SU i bids p_i and v_i, respectively. We first consider the scenario where $p_i > v_i$.

- **Case 1**: *SU i loses with both v_i and p_i.* In this case, $u_i(p_i, \mathbf{p_{-i}}) = u_i(v_i, \mathbf{p_{-i}}) = 0$ according to our definition in (1). Thus, (2) holds.
- **Case 2**: *SU i loses with v_i but wins with p_i.* In this case, obviously we have $u_i(v_i, \mathbf{p_{-i}}) = 0$. Since SU i wins with p_i, in an arbitrary t-th iteration ($t \geq T_0$), we can obtain

$$\begin{aligned} & u_i(p_i, \mathbf{p_{-i}}) \\ = & (v_i - c_i) \sum_{q=1}^{t} \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q}) \\ = & v_i \sum_{q=1}^{t} \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q}) - \sum_{q=1}^{t} \left(\frac{b_i^q}{B_q} \cdot C_t \right) \\ = & \sum_{q=1}^{t} \left(v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t \right), \end{aligned}$$

where $b_{-i}^q = \sum_{j \in \mathcal{G}_q, j \neq i} b_j^q$. Besides, since SU i is a winning SU, there must be at least one out of the t winning VBGs that contains i. Denote the set of the indexes of such VBGs by H. Then, we get $v_i^q = b_i^q = 0$ for $q \notin H$, and hence

$$u_i(p_i, \mathbf{p_{-i}}) = \sum_{q \in H} \left(v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t \right).$$

In addition, since $v_i < p_i$, we can have that for any $q \in H$, $v_i^q < b_i^q$ and thus

$$v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t < v_i^q - \frac{v_i^q}{b_{-i}^q + v_i^q} C_t. \qquad (17)$$

Furthermore, for any winning VBG that contains SU i, say $\mathcal{G}_q$ ($q \in H$), its virtual bid satisfies $B_q = b_{-i}^q + b_i^q \geq C_t$. Due to the fact SU i loses by bidding v_i, the t winning VBGs, denoted by $\overline{\mathcal{G}}_k$ ($1 \leq k \leq t$) with virtual bid $\overline{B}_k$, do not contain SU i when SU i bids v_i. Thus, we have $b_{-i}^q + v_i^q \leq \overline{B}_t$. Since at least one VBG containing SU i becomes a winner when SU i bids p_i, $\overline{\mathcal{G}}_t$ must lose, and hence $\overline{B}_t \leq C_t$. Thus, we have $C_t \geq b_{-i}^q + v_i^q$ for $q \in H$. As a result, we finally get

$$u_i(p_i, \mathbf{p_{-i}}) < \sum_{q \in H} (v_i^q - \frac{v_i^q}{b_{-i}^q + v_i^q} C_t) < 0,$$

which leads to $u_i(p_i, \mathbf{p_{-i}}) \leq u_i(v_i, \mathbf{p_{-i}})$ as well.
- **Case 3**: *SU i wins with v_i and loses with p_i.* Since $p_i > v_i$, according to the monotonicity property we have proved in Lemma 3, this will not happen.
- **Case 4**: *SU i wins with both v_i and p_i.* In an arbitrary t-th iteration ($t \geq T_0$), we denote the set of the indexes of the winning VBGs containing SU i when SU i bids with p_i and that when U i bids with v_i by H and H', respectively. We also denote the clearing prices when i bids with p_i and v_i by C_t and C'_t, respectively. Notice that 1) if the set of winning VBGs when SU i bids with p_i and that when SU i bids with v_i, denoted by $\mathcal{G}_W^q(p_i)$ and $\mathcal{G}_W^q(v_i)$, respectively, are the same, we have $C_t \geq C'_t$ according to (17) since the VBGs' virtual bids are larger when SU i bids with p_i and λ_q's remain the same; 2) if $\mathcal{G}_W^q(p_i)$ and $\mathcal{G}_W^q(v_i)$ are different, it means at least one of the winning VBGs when SU i bids with v_i loses when SU i bids with p_i. Since this VBG's virtual bid when SU i bids with p_i, denoted by B_x, is no smaller than that when when SU i bids with v_i, denoted by B'_x, we have $C_t \geq B_x \geq B'_x \geq C'_t$. Consequently, we always have $C_t \geq C'_t$. Besides, similar to that in Case 2, we get

$$\begin{aligned} u_i(p_i, \mathbf{p_{-i}}) &= \sum_{q \in H} \left(v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t \right) \\ u_i(v_i, \mathbf{p_{-i}}) &= \sum_{q \in H'} \left(v_i^q - \frac{v_i^q}{b_{-i}^q + v_i^q} C'_t \right) \end{aligned}$$

When SU i bids with p_i, denote its utility attributed to the common VBGs between $\mathcal{G}_W^q(p_i)$ and $\mathcal{G}_W^q(v_i)$ by $u_i^1(p_i, \mathbf{p_{-i}})$ and the utility attributed to the other VBGs by $u_i^2(p_i, \mathbf{p_{-i}})$. Similarly, when SU i bids with v_i, denote its utility attributed to the common VBGs between $\mathcal{G}_W^q(p_i)$ and $\mathcal{G}_W^q(v_i)$ by $u_i^1(v_i, \mathbf{p_{-i}})$ and the utility attributed to the other VBGs by $u_i^2(v_i, \mathbf{p_{-i}})$. Then, we have the following results.

First, for those common VBGs between $\mathcal{G}_W^q(p_i)$ and $\mathcal{G}_W^q(v_i)$, we have

$$\begin{aligned} & u_i^1(p_i, \mathbf{p_{-i}}) - u_i^1(v_i, \mathbf{p_{-i}}) \\ \leq & \sum_{q \in (H \cap H')} \left(v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t \right) \\ & - \sum_{q \in (H \cap H')} \left(v_i^q - \frac{v_i^q}{b_{-i}^q + v_i^q} C_t \right) \end{aligned}$$

which is less than 0 according to (17).

Second, for any VBG in $\mathcal{G}_W^q(p_i)$ but not in $\mathcal{G}_W^q(v_i)$, we have

$$v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t < v_i^q - \frac{v_i^q}{b_{-i}^q + v_i^q} C_t.$$

Since this VBG loses when SU i bids with v_i, we have $C_t \geq C'_t \geq b_{-i}^q + v_i^q$. Thus, we get

$$u_i^2(p_i, \mathbf{p_{-i}}) = \sum_{q \in H \setminus (H \cap H')} \left(v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t \right) \leq 0.$$

Third, for any VBG in $\mathcal{G}_W^q(v_i)$ but not in $\mathcal{G}_W^q(p_i)$, we have $v_i^q - \frac{v_i^q}{b_{-i}^q + v_i^q} C'_t \geq 0$ since this VBG wins when SU i bids with v_i and $C'_t \leq b_{-i}^q + v_i^q$. Thus, we obtain

$$u_i^2(v_i, \mathbf{p_{-i}}) = \sum_{q \in H' \setminus (H \cap H')} \left(v_i^q - \frac{b_i^q}{b_{-i}^q + b_i^q} C_t \right) \geq 0.$$

As a result, we can get

$$\begin{aligned}&u_i(p_i, \mathbf{p_{-i}}) - u_i(v_i, \mathbf{p_{-i}})\\ =&u_i^1(p_i, \mathbf{p_{-i}}) - u_i^1(v_i, \mathbf{p_{-i}}) + u_i^2(p_i, \mathbf{p_{-i}}) - u_i^2(v_i, \mathbf{p_{-i}})\\ \leq&0.\end{aligned}$$

The proof is similar when $p_i < v_i$, which is omitted due to space limit.

In general, $u_i(p_i, \mathbf{p_{-i}}) \leq u_i(v_i, \mathbf{p_{-i}})$ always holds, and hence the theorem directly follows. ■

Theorem 3: The proposed auction scheme TOA is IR for SUs.

Proof: In an arbitrary t-th iteration ($t \geq T_0$), since TOA is IR for VBGs, we have $C_t \leq B_q$ for $1 \leq q \leq t$, and hence

$$\begin{aligned} c_i &= \frac{\sum_{q=1}^{t}\left(\frac{b_i^q}{B_q}\cdot C_t\right)}{\sum_{q=1}^{t}\sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q})}\\ &\leq \frac{\sum_{q=1}^{t} b_i^q}{\sum_{q=1}^{t}\sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q})}\\ &= \frac{\sum_{q=1}^{t} p_i \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q})}{\sum_{q=1}^{t}\sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{T}_i^m} c_{ij}^m(\mathcal{I}_{W,q})} = p_i. \end{aligned}$$

Therefore, TOA is IR for SUs. ■

From Theorem 2 and Theorem 3, we can have the following theorem.

Theorem 4: The proposed auction scheme TOA is economic-robust for SUs.

VI. Simulation Results

In this section, we conduct simulations to evaluate the performance of our proposed auction scheme TOA. Simulations are carried out in CPLEX 12.4 on a computer with a 2.27 GHz CPU and 24 GB RAM. We randomly deploy SUs in a square network of area $1000m \times 1000m$. There are totally 5 multi-hop sessions in the network, each of which has traffic demand of 1Mbps. We assume that each bidder's true valuation of (and hence its bid for) unit instantaneous transmission rate is uniformly distributed over $[10^{-6}, 10^{-5}]$. In addition, assume the PU has 3 idle spectrum bands to lease to the SUs, with their bandwidths being 1.0MHz, 1.5MHz and 2.0MHz, respectively. Some other important simulation parameters are listed as follows. The path loss exponent is 4 and $C = 62.5$. The noise power spectral density is $\eta = 3.34 \times 10^{-20}$W/Hz at all nodes. The transmission power spectral density of nodes is $8.1\times10^7\eta$, and the reception threshold and interference threshold are both 8.1η on each spectrum band. Thus, the transmission range and the interference range on each frequency band are both equal to 500m. Since we have proved that our auction scheme is economic-robust in the previous section, we demonstrate the *auction efficiency* and the auctioneer's revenue in what follows. Note that auction efficiency is defined as the ratio of the number of finally successfully delivered traffic flows to the total number of traffic flows demanded by the SUs.

We first compare the auction efficiency of the proposed TOA scheme with those of two other auction schemes: one for single-hop data transmission [10], and the other for multi-hop data transmission [12] which greedily assigns spectrum bands to links. We call these two schems 1-hop auction and greedy multi-hop auction, respectively, in our simulations. To make fair comparisons, we compare TOA with these two schemes in single-hop and multi-hop scenarios, respectively.

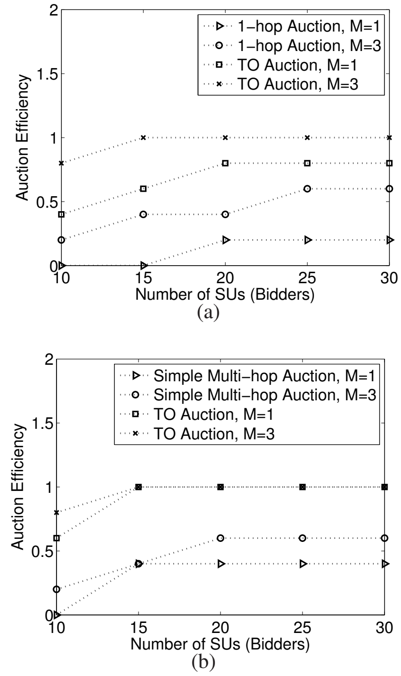

Fig. 1. Auction efficiency comparison with 1-hop auction scheme and greedy multi-hop auction scheme. (a) Single-hop data transmission scenario. (b) Multi-hop data transmission scenario.

In the single-hop scenario, each source SU can reach its intended destination SU in one hop, and hence the data traffic can be delivered in one-hop as well. Fig. 1(a) gives the results when the number of SUs ranges from 10 to 30 and the number of available spectrums M is equal to 1 and 3. We can find that TOA can achieve much higher auction efficiency than 1-hop auction. Particularly, in the case that there is only one available spectrum band, TOA can support two and three traffic flows when the number of SUs is 10 and 15, respectively, while 1-hop auction cannot support any of the traffic flows. When there are more SUs in the network, TOA can support four traffic flows while 1-hop auction can only support one of them. In the case that there are three available spectrum bands, TOA can support four flows when there are 10 SUs and all the five flows when there are more SUs, while 1-hop auction can only support one flow, two flows, and three flows, when there are 10, 15 and 20, and more SUs, respectively. As we mentioned before, this is because in 1-hop auction, it is not clear whom a winning SU communicates with and there can be a lot of collisions in the network.

In the multi-hop scenario, each source node needs to deliver data to its destination via multiple hops. The auction efficiency is shown in Fig. 1(b) when the number of SUs ranges from 10 to 30 and the number of available spectrums M is equal to 1 and 3. In particular, in the case that there is only one available spectrum band, TOA can support three traffic flows when the number of SUs is 10, and all the five traffic flows when there are more SUs in the network. On the other hand, greedy multi-hop auction cannot support any traffic flows when there are 10 SUs, and only two flows when there are more SUs. Besides, in the case that there are three available spectrum bands, TOA can support four flows when there are 10 SUs and five flows when there are more SUs, while greedy multi-hop auction can only support one flow, two flows, and three flows, when there are 10, 15, and more SUs, respectively. This is because that we consider transmission opportunities in auctions as well as spectrum scheduling in both frequency and time domains.

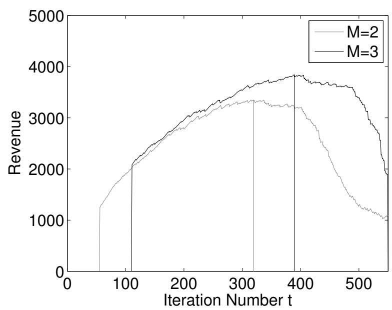

Fig. 2. Computation performance under different number of spectrum band.

We then illustrate the revenues TOA generates for the auctioneer. Note that in TOA, SUs bid for TOs based on unit instantaneous transmission rate, while the other auction schemes bid for spectrum bands based on bandwidth. Thus, SUs' valuations and bids have very different meanings from those in previous schemes, and we cannot compare with their revenues here. In this case, we consider the scenarios where there are 20 SUs and the number of available spectrum bands M is equal to 2 or 3. The results are shown in Fig. 2. We can see that the auctioneer's revenue first grows and then declines as the iteration number increases. We can also find that the number of iterations does not need to be very large. For example, when $M = 2$, T_0 is around 55, and the maximum revenue is achieved when the iteration number is approximately 310. When $M = 4$, the maximum revenue is achieved when the iteration number is about 390. Moreover, the auctioneer's revenue is higher when there are more spectrum bands, which fits our intuition since it sells more resources. On the other hand, the revenue when $M = 3$ does not exceed much than that when $M = 2$ since the auctioneer does not need to fully utilize all the spectrum resources to support the traffic and can save some for other applications.

VII. Conclusions

In this paper, we have proposed a novel spectrum auction scheme, called transmission opportunity auction (TOA), based on TOs. The TOA scheme is mainly composed of three procedures: TO allocation, TO scheduling, and pricing. In TO allocation, in each iteration the auctioneer finds out the VBG that has the highest virtual bid. In TO scheduling, the auctioneer checks if the winning VBGs found so far can support the traffic demand in the network by solving a minimum length scheduling problem. In pricing, the auctioneer determines the clearing price for each winning VBG and SU, and computes its own revenue. The auctioneer finally chooses the winning VBGs which can generate the highest revenue among the results it obtains. We have proved that TOA is IC, IR, and BB, and hence economic-robust. We have also carried out extensive simulations which show that TOA leads to high spectrum utilization and efficiently generates high profits for the auctioneer.

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